Efficient nonlinear equalizer for intra-channel nonlinearity compensation for next generation agile and dynamically reconfigurable optical networks

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Abstract: In this work, we propose and experimentally demonstrate a novel low-complexity technique for fiber nonlinearity compensation. We achieved a transmission distance of 2818 km for a 32-GBaud dual-polarization 16QAM signal. For efficient implantation, and to facilitate integration with conventional digital signal processing (DSP) approaches, we independently compensate fiber nonlinearities after linear impairment equalization. Therefore this algorithm can be easily implemented in currently deployed transmission systems after using linear DSP. The proposed equalizer operates at one sample per symbol and requires only one computation step. The structure of the algorithm is based on a first-order perturbation model with quantized perturbation coefficients. Also, it does not require any prior calculation or detailed knowledge of the transmission system. We identified common symmetries between perturbation coefficients to avoid duplicate and unnecessary operations. In addition, we use only a few adaptive filter coefficients by grouping multiple nonlinear terms and dedicating only one adaptive nonlinear filter coefficient to each group. Finally, the complexity of the proposed algorithm is lower than previously studied nonlinear equalizers by more than one order of magnitude.

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OCIS codes: (060.2330) Fiber optics communications; (060.4230) Multiplexing; (060.1660) Coherent communications.

References and links
1. Introduction

To satisfy the ever-increasing capacity demand in optical fiber communications, both the spectral efficiency (SE) and the data-rate carried by each wavelength division multiplexed channel has to increase. According to Shannon’s theory of linear communication systems, the channel capacity is logarithmically proportional to signal-to-noise ratio (SNR). Therefore, the capacity can be increased by increasing the signal power. However, because of fiber Kerr nonlinearities, there is an optimum launch power limit. Further increases in input signal power beyond the optimal power levels degrades transmission performance. Consequently, fiber nonlinearities are the major remaining impairments for the next generation coherent optical fiber communication systems that ultimately limit the achievable transmission distance [1].

Following recent advances in high-speed digital signal processing (DSP) technology, along with global adoption of coherent detection techniques, various intra-channel fiber nonlinearity compensation algorithms have been proposed [2]. For instance, digital back-propagation is an effective nonlinear compensation (NLC) technique, which has received considerable attention [3–5]. It normally requires multiple computation steps per each fiber span and at least two samples per symbol. This leads to high complexity [6]. Consequently, its application is limited to the transmission systems using offline signal processing.

There are also reports of nonlinear frequency-domain equalizers based on closed form analytical approximations of fiber third-order Volterra kernels. These techniques simultaneously compensate both nonlinear and linear impairments. However, their major
drawback is their high complexity [7,8]. A Wiener-Hammerstein equalizer (for OFDM transmission) [9] and modified nonlinear decision feedback equalizer [10] was proposed as a simpler alternative. These algorithms adaptively compensate for all fiber impairments according to following formulas (presented in time-domain):

\[
A_{\text{Out}}^{x,y} = \frac{1}{n_L} \sum_{l=-n_L}^{n_L} H_{1,xx} A_{x,x} + \frac{1}{n_L} \sum_{l=-n_L}^{n_L} H_{1,yy} A_{x,y} + \sum_{m,n,k=-n_{NL}}^{n_{NL}} C_{m,n,k} A_{x,x}^* A_{x,x} + \sum_{m,n,k=-n_{NL}}^{n_{NL}} C_{m,n,k} A_{x,y}^* A_{x,y} + \sum_{m,n,k=-n_{NL}}^{n_{NL}} C_{m,n,k} A_{x,y}^* A_{x,y}
\]

(1)

\[
A_{\text{Out}}^{x,y} = \frac{1}{n_L} \sum_{l=-n_L}^{n_L} H_{1,yy} A_{x,x} + \frac{1}{n_L} \sum_{l=-n_L}^{n_L} H_{1,xx} A_{x,y} + \sum_{m,n,k,-n_{NL}}^{n_{NL}} C_{m,n,k} A_{x,y}^* A_{x,y} + \sum_{m,n,k,-n_{NL}}^{n_{NL}} C_{m,n,k} A_{x,y}^* A_{x,y} + \sum_{m,n,k,-n_{NL}}^{n_{NL}} C_{m,n,k} A_{x,y}^* A_{x,y}
\]

(2)

where \(n_L\) and \(n_{NL}\) are linear and nonlinear equalizer memory lengths, respectively. Their nonlinear filter computational complexity grows as \(n_{NL}^3\) and they require a long training sequence for their \(4 \cdot n_{NL}^3\) adaptive nonlinear filter coefficients. Therefore, their performance was investigated for single polarization systems with inline dispersion compensation and a short channel nonlinear memory length (i.e. fiber with small dispersion parameter) [9,10].

The perturbation-based nonlinear pre-compensation (or post-equalization) technique compensates the accumulated nonlinearities with only one computation step and can be implemented with one sample per symbol [11–13]. Typically, perturbation based NLCs have lower computational complexity than blind nonlinear equalization algorithms [7–10]. However, perturbation based NLC algorithms requires prior knowledge of fiber optic transmission system parameters in order to calculate perturbation coefficients [11] and 50% chromatic dispersion (CD) pre-compensation at the transmitter for efficient implementation [12,13]. Development of wavelength-selective switching (WSS) technologies and flexible optical transceivers (with reconfigurable rates and modulation formats) is enabling the next generation of transparent optical networks. Remote reconfiguration of a mesh network (i.e. dynamic networking) and adaptation of the flexible optical transceivers provides optimal network utilization and agility [14,15]. Thus, many sets of perturbation coefficients (depending on different transceiver configuration, CD pre-compensation and selected transmission route) have to be stored in the line card memory. In addition, identification of correct set of perturbation coefficients becomes a very difficult task once all possible network parameters are considered. In addition, for certain scenarios, the exact transmission route is unknown at the transmitter or receiver. Therefore, perturbation based NLC cannot be easily deployed in dynamically reconfigurable meshed optical network architectures and its application is restricted to point-to-point systems with well-studied a parameters.

Low-complexity adaptive nonlinear equalization algorithms are highly desirable for next generation agile and flexible high-data rate fiber optic communication systems. We propose a novel nonlinear equalizer based on the first-order perturbation model with quantized perturbation coefficients. The proposed equalizer deals with the nonlinear impairment only and can be implemented in any single-carrier communication receiver after conventional single carrier DSP [16]. Its computational complexity is similar to the perturbation based NLCs [11,12], but on the other hand it does not require accurate knowledge of transmission link and transceiver parameters. Here, we adopted the decision directed least mean square (DD-LMS) algorithm for learning and optimum adaptation of nonlinearity coefficients at the receiver.
This paper is organized as follows: in Section 2, we review perturbation based NLC algorithms. Section 3 introduces our nonlinear equalization scheme. Section 4 describes the experimental setup used to evaluate the performance of the proposed algorithms. Section 5 then discuss the experimental results. Finally, Section 6 presents our conclusions.

2. Motivation and principles of perturbation based nonlinearity compensation

The evolution of the optical field envelope in a fiber optic link is described by the nonlinear Schrödinger equation (NLSE) [17]:

\[
\frac{\partial}{\partial z} u(t, z) + j \frac{\beta_2(z)}{2} \frac{\partial^2}{\partial t^2} u(t, z) = j \gamma(z) \left| u(t, z) \right|^2 u(t, z)
\]  

where \( u(t, z) \) is the optical field, \( \beta_2(z) \) is the group velocity dispersion, and \( \gamma(z) \) is the nonlinear coefficient. The nonlinear term in Eq. (3) can be treated as a small perturbation by denoting \( u(t, z) = u_0(t, z) + j \Delta u(t, z) \), where \( u_0(t, z) \) is the solution of linear propagation and \( \Delta u(t, z) \) is the perturbation due to the nonlinear effects. Under the first order approximation and based on phase-matching condition, among all possible symbol triplets (with indices \( m, n \) and \( k \)) that cause intra-channel nonlinear impairments, only the triplets that hold the \( k = m + n \) property play a significant role. Therefore, the intra-channel four-wave-mixing (IFWM) and intra-channel cross-phase-modulation (IXPM) nonlinear-induced distortions on the transmitted symbol can be expressed as (we have removed the triplets, which result in phase rotation) [7]:

\[
\Delta A_{0,x} = P^{1/2} \left[ \sum_{m,n=0}^{k} C_{m,n} A_{m,x} A_{m+n,0} A_{0,x} + \sum_{m,n=0}^{k} C_{m,n} A_{m,y} A_{m+n,x} A_{0,y} \right]
\]

\[
\Delta A_{0,y} = P^{1/2} \left[ \sum_{m,n=0}^{k} C_{m,n} A_{m,y} A_{m+n,x} A_{0,x} + \sum_{m,n=0}^{k} C_{m,n} A_{m,x} A_{m+n,y} A_{0,y} \right]
\]

Here \( x \) and \( y \) subscripts denote the two polarizations, \( P \) is the optical signal power, \( C_{m,n} \) is the perturbation coefficient with \( m \) and \( n \) denoting the symbol index relative to the current symbol. \( A_{m,x/y} \) is the transmitted symbol. Equations (4) and (5) imply that the nonlinear field is a linear combination of triplets consisting of transmitted symbols, weighted by \( C_{m,n} \) coefficients. Perturbation coefficients can be numerically calculated as follow [8]:

\[
C_{m,n} = i k \int_0^\infty dz \gamma(z) f(z) \int dt g^{(0)}(z,t) g^{(0)}(z,t-mT) g^{(0)}(z,t-nT) g^{(0)}(z,t-(n+m)T). \]

Here \( \gamma(z) \) denotes the fiber nonlinear coefficient, \( k \) is the scaling factor \( f(z) \) describes the power distribution profile along the link, \( T \) stands for the symbol period, \( g^{(0)}(0,t) \) is the pulse shape with zero accumulated dispersion \( (z = 0) \), and \( g^{(0)}(z,t) \) is the dispersed pulse shape corresponding to a fiber length \( z \) which is calculated according to

\[
g^{(0)}(z,t) = \text{iff} \left[ \text{fft} \left[ g^{(0)}(0,t) \right] \times \exp \left[ -i \beta_2(2\pi f)^2 z / 2 \right] \right].
\]

(\text{iff}) denotes the (inverse) Fourier transform, \( f \) is frequency, \( \beta_2 \) and is the first-order group velocity dispersion [8]. Figure 1 demonstrates the normalized perturbation coefficients after 480 km of single-mode optical fiber (SMF).
Assuming Gaussian pulses and ignoring the attenuation, analytical expressions in terms of the exponential integral function exist for the nonlinear coefficients [11]:

\[
C_{m,n} = \begin{cases} 8 \frac{\gamma^2}{9 \sqrt{3} |\beta_2|^2} E_i \left( \frac{(m-n)^2 T^2 \tau^2}{3 |\beta_2|^2 L^2} \right) & \text{m or n} = 0 \\
8 \frac{\gamma^2}{9 \sqrt{3} |\beta_2|^2} E_i \left( -j \frac{mnt^2}{\beta_2 L} \right) & \text{m,n} \neq 0 \end{cases}
\]  

(8)

where \( \tau, T \) and \( L \) are the pulse-width, the inverse of symbol rate and the transmission distance, respectively. \( m \) and \( n \) are the symbol indices, and \( E_i(\cdot) \) is the exponential integral function [18]. It can be seen that for \( \forall m,n \neq 0 \),

\[
C_{m,n} = C_{n,m} = C_{-m,-n} = C_{-n,-m} = 0
\]

(10)

\[
C_{m,n} = C_{n,m} = -C_{-m,-n} = -C_{-n,-m}
\]

(11)

It is evident that, \( C_{m,n} \) varies with only a single parameter \( q = m \cdot n \) and thus, all pairs of \((m,n)\) could share the same coefficient \( C_q \), as long as their product equals a unique \( q \). It should be noted that in the case of other symmetric pulses the perturbation coefficients still satisfy the properties stated in Eqs. (10) and (11). We also verified Eqs. (10) and (11) by numerical integration of Eq. (6) for a root raised cosine (RRC) pulse shaping filter.

For more efficient implementation of the algorithm, quantization of the coefficients (i.e. combining multiple terms with similar perturbation coefficients) can be used to reduce the number of complex multiplications [12,13]. However, there is an expected trade-off: fewer quantization levels lead to lower complexity, at the expense of reduced performance. In this case, Eqs. (4) and (5) can be simplify as:

\[
\Delta A_{y,x} = P^{1/2} \sum_{k=1}^{N} C_k \left( \sum_{\forall m,n,C_y} A_{n,x} A'_{n+1,x} A_{m,x} + \sum_{\forall m,n,C_y} A_{m,y} A'_{m+1,y} A_{m,x} \right)
\]  

(12)
\[ \Delta A_{n,y} = P^{1/2} \sum_{k=1}^{N_k} C_k \left( \sum_{\gamma,m \in C_k} A_{n,y} A'_{\gamma+m,n,y} A_{m,y} + \sum_{\gamma,m \in C_k} A_{n,x} A'_{\gamma+m,n,x} A_{m,y} \right) \]  

(13)

where \( N_k \) is the total number of quantization levels and the region representative \( C_k \), was obtained using this formula:

\[ m, n \in C_k \quad \text{if} \quad C_{k,\text{lower}} \leq C_{m,n} \leq C_{k,\text{upper}} \]

\[ C_k = \frac{C_{k,\text{upper}} + C_{k,\text{lower}}}{2} \]  

(14)

Unfortunately, conventional uniform quantization does not provide optimum performance [18] and optimized quantization levels and quantized values, \( C' \), have to be determined. The exhaustive search method with minimum mean square error (MMSE) as criteria for the level estimation has been proposed for offline calculation of optimum quantized perturbation coefficients and the corresponding quantization levels [19].

In cases where a single fiber type is deployed throughout the transmission path, and assuming a symmetric power profile and dispersion map (which can be readily obtained by 50% CD pre-compensation at the transmitter), it can be shown that perturbation coefficients become imaginary-valued [12,13]. This reduces the computational complexity by replacing the complex multipliers with real multipliers and by reducing channel nonlinear memory. However, this approach has a limited practical use when it comes to reconfigurable mesh optical networks. In addition, most of the fibers in optical networks come from the legacy networks, which contain different fiber types (ITU-T G.652, G.653, and G.655) with widely varying characteristics [20]. Therefore, a symmetric dispersion map cannot be obtained by only performing 50% CD pre-compensation at the transmitter. In addition, for long haul fiber optic transmissions CD pre-compensation would largely enhance the signal’s peak-to-average power ratio (PAPR), which would inevitably increase DAC quantization and clipping noises [21], in addition to enhancing the nonlinear impairment.

3. Decision directed least mean square (DD-LMS) nonlinear filter equalizer

Assuming linear impairments are compensated by conventional single carrier DSP [16], details of proposed equalizer are as follows. After rephrasing Eqs. (1) and (2) with respect to Eqs. (12) and (13), output of the nonlinear equalizer can be express as:

\[ A'_{x} = A_x + C_x \Gamma_{x,1} + C_x \Gamma_{x,2} + \cdots + C_x' \Gamma'_{x,1} + C_x' \Gamma'_{x,2} + \cdots \]  

(15)

\[ A'_{y} = A_y + C_y \Gamma_{y,1} + C_y \Gamma_{y,2} + \cdots + C_y' \Gamma'_{y,1} + C_y' \Gamma'_{y,2} + \cdots \]  

(16)

where we define:

\[ \Gamma_{x,k} = \sum_{m,n \in C_k} A_{n,x} A'_{m+n,x} A_{m,x} + A_{n,y} A'_{m+n,y} A_{m,y} \quad & \Gamma'_{x,k} = \sum_{m,n \in C_k} A_{n,x} A'_{m+n,x} A_{m,y} \]  

(17)

\[ \Gamma_{y,k} = \sum_{m,n \in C_k} A_{n,y} A'_{m+n,y} A_{m,x} + A_{n,x} A'_{m+n,x} A_{m,y} \quad & \Gamma'_{y,k} = \sum_{m,n \in C_k} A_{n,y} A'_{m+n,y} A_{m,y} \]  

(18)

Equations (15) and (16) imply that the nonlinear equalizer output can be express as a linear combination of symbol triplet sums i.e. \( \Gamma_{x,y,k} \) \( (m,n \neq 0) \) and \( \Gamma'_{x,y,k} \) \( (n = 0) \). The motivation to split the sums into two different linear combinations of \( \Gamma_{x,y,k} \) and \( \Gamma'_{x,y,k} \) lies in the fact that \( C_k' \) is always purely imaginary according to Eq. (6), whereas \( C_k' \) is generally a complex number. Based on adaptive filter theory, each \( C_k \) and \( C_k' \) can be learned and
continuously updated utilizing the stochastic gradient algorithm using the following set of equations:

\[ C_{kx,y}(p+1) = C_t(p) + \mu \varepsilon_{kx,y}(p) \Gamma_{x,y,k}(k) \]  \hspace{1cm} (19)

\[ C'_{kx,y}(p+1) = C'_t(p) + i \cdot \text{imag} \{ \mu \varepsilon_{kx,y}(p) \Gamma''_{x,y,k}(k) \} \]  \hspace{1cm} (20)

Here, \( \mu \) is the algorithm step size, \( p \) denotes the update step index and \( \varepsilon_{kx,y} \) is the error at \( p \)-th step, given by

\[ \varepsilon_{x,y}(p) = A_{x,y} \cdot \text{decision} \{ A_{x,y}(p) \} \]. \hspace{1cm} (21)

\( C_t \) and \( C'_t \) are the same for both polarizations. Therefore, Eqs. (19) and (20) can be implemented over both polarizations independently. Averaging coefficients over two polarizations will result in a better estimation and higher noise rejection. Alternatively, for a more efficient implementation, the adaptation process can be divided between two polarizations where half of the coefficients are updated using x-polarization, and the remaining coefficients are calculated using y-polarization. In this paper, the later approach has been used.

In order to identify indices of triplet symbols, which constitute the triplet sums (i.e. \( \Gamma_{x,y,k} \) or \( \Gamma'_{x,y,k} \)), the nonlinear channel memory length has to be determined first. Pulse broadening induced by chromatic dispersion leads to multiple pulse collisions in an optical fiber. These pulses interact with each other due to fiber Kerr effect and induce nonlinear distortion on the transmitted symbols. Therefore, maximum nonlinear memory of the fiber is highly related to the CD-induced pulse broadening (normalized by symbol length) expressed by [16]

\[ n_{\text{CD}}^\text{max} = \frac{\Delta T}{T} = \frac{B_w \frac{c}{f_0} \int_{L_{\text{tot}}/2}^{L_{\text{tot}}} D(z) \, dz}{T} \]  \hspace{1cm} (22)

Here, \( D(z) \) and \( L_{\text{tot}} \) denote the dispersion parameters and total link length, respectively. \( B_w, T, f_0, \) and \( c \) are the bandwidth of signal, symbol duration, center frequency of channel of interest and speed of light, respectively.

At the launch point, accumulated dispersion and channel nonlinear memory are equal to zero. However, as a pulse propagates down the fiber, CD increases the linear and nonlinear channel memory. At the receiver, it is equals to \( n_{\text{CD}}^\text{max} \). Therefore, we use half of the maximum pulse broadening as an approximation for effective fiber nonlinearity memory i.e.

\[ n_{\text{eff}}^\text{max} = \left\lfloor n_{\text{CD}}^\text{max}/2 \right\rfloor \]. \hspace{1cm} (23)

\( \left\lfloor \cdot \right\rfloor \) denotes the floor operator. We point out that accumulated dispersion can be easily extracted from the CD compensation equalizer or any CD monitoring algorithms [22]. In addition, fiber nonlinearity memory can be set manually based on acceptable computational complexity and available DSP resources.

Based on properties extracted from Eqs. (10)-(14), we uniformly group symbols into \( N_1 + 2N_2 \) sets of indices ( \( N_1 \) sets for \( n = 0 \) corresponding to \( C_t \) and \( 2N_2 \) sets for \( m, n \neq 0 \) corresponding to \( C'_t \) indices as follow:

\[ \tau_i = \left\{ (m,0) \mid m \in \mathbb{Z} - \{0\} \right\} \text{ and } m_i \leq |m| < m_{i+1}, \hspace{0.5cm} m_i = \left\lfloor i \cdot \frac{n_{\text{eff}}^\text{max}}{N_1} \right\rfloor \text{ and } i = 0,1,\ldots,N_1 \right\} \]  \hspace{1cm} (24)
\[ \tau'_i = \begin{cases} (m,n) & m, n \in \mathbb{Z} - \{0\} \text{ and } q_i \leq m \cdot n < q_{i+1} \\ q_i = \left[ \frac{i \cdot n_{\text{eff}}^N}{N_2^2} \right] & i = -N_2, -N_2 + 1, \ldots, N_2 \end{cases} \]

(25)

\[ |\cdot| \] denotes the absolute value operator. Notice that, for \( \tau'_i \), in accordance with Eq. (9) the product of \( m \) and \( n \) (i.e., \( q = m \cdot n \)) is used for partitioning the symbols. The maximum value for \( q \) is \( n_{\text{eff}}^N \). Also, for \( \tau'_i \) and \( i = -N_2, -N_2 + 1, \ldots, 0 \) we have \( m \cdot n < 0 \) which corresponds to triplets with the first two indices in 1st and 3rd quadrature whereas \( i = 0, 1, \ldots, N_2 \) results in \( m \cdot n > 0 \) and similar sets of indices in 2nd and 4th quadrature. In this case and based on Eq. (11), for the corresponding perturbation coefficients, we have \( C'_k = -C''_{N_k} \). We used this property and averaging to improve estimates of the perturbation coefficients.

Figure 2 shows all symbol indices that have been used in our nonlinearity compensation algorithm and the perturbation-based NLC when \( \max \{|C_{m,n}|, |C_{0,0}|\} < -35 \text{ dB} \). We observe that the proposed algorithm uses similar indices for adaptive perturbation nonlinearity compensation. Furthermore, we numerically integrated Eq. (6) for different system parameters, and observed that in all cases our nonlinear equalizer uses smaller set of triplet’s indices in comparison to the perturbation-based NLCS [12]. Therefore, the proposed algorithm should have similar computational complexity to the perturbation-based NLCS [11,12].

![Fig. 2. Symbol indices for the perturbation-based nonlinear compensation and the proposed adaptive nonlinear equalizer after 480 km of SMF.](image)

We calculated the total number of total symbol triplets for our nonlinear equalizer as follow:

\[
4 \cdot \sum_{m=1}^{N_2} \left[ \frac{(n_{\text{eff}}^N - 1)}{n} \right] + 2n_{\text{eff}}^N - 1 = (n_{\text{eff}}^N - 1) \cdot \log(n_{\text{eff}}^N - 1) + 3n_{\text{eff}}^N - 1. \tag{26}
\]

In comparison to previously studied nonlinear equalizers [10–13], the total number of triplet indices is reduced from \( (n_{\text{eff}}^N)^3 \) to \( (n_{\text{eff}}^N - 1) \cdot \log(n_{\text{eff}}^N - 1) \), which is smaller by more than one order of magnitude.
4. Experimental setup

Figure 3 shows the schematic diagram of the experimental setup. On the transmitter side, offline DSP, four 2-tuple independent pseudo-random bit sequences (PRBS) are mapped to 16QAM symbols, followed by pulse shaping at 2 samples per symbol for each polarization. A Ciena WaveLogic 3 (WL3) line card was employed, which contains four 39.5 GSa/s 6 bit digital-to-analog converter (DACs), a tunable frequency laser source, and a dual-polarization (DP) IQ-modulator. The transmitter laser was operating at 1554.94 nm. The transmitter analog frequency response is compensated using the on-board built-in DSP of the WL3. The output optical signal is then boosted to 23 dBm using an erbium-doped fiber amplifier (EDFA), and subsequently attenuated using a conventional variable optical attenuator (VOA) in order to get a desired optical launch power. The optical signal is then launched into a recirculating loop. This loop consists of four spans of 80 km of single mode fiber (SMF-28e + LL) and four inline EDFAs. Each inline EDFA has a noise figure of 5.5 dB. A tunable bandwidth and tunable center wavelength band-pass filter (T-T BPF) is inserted after the 4th span in order to reject out-of-band amplified spontaneous emission (ASE) noise accumulated during transmission. The gain of the last EDFA is adjusted (increased by 10 dB compared to the other EDFAs) in order to compensate for losses occurring inside the recirculating loop, including switches, couplers and the band-pass filter.

At the receiver side, an optical spectrum analyzer (OSA) was used in order to measure the signal optical signal-to-noise ratio (OSNR) at 0.5 nm resolution bandwidth and then it was converted to a 0.1 nm noise bandwidth. The gain of the pre-amplifier EDFA was adjusted to ensure that the signal power reaching the coherent receiver was held constant at 5 dBm. Finally, a 0.8 nm BPF was used to filter out the out-of-band amplified spontaneous emission noise generated by the pre-amplifier. At the polarization-diversity 90° optical hybrid, the signal was mixed with 15.5 dBm local oscillator (LO) light from an external-cavity laser (ECL) with a linewidth of 100 kHz. The beating outputs were passed through four balanced photodetectors. A 4-channel real-time oscilloscope sampled the signal at a sampling rate of 80 GSa/s and digitized it with 8-bit resolution.

Figure 4 shows the top-level block diagram of the receiver. The DSP code starts with front-end compensation, including the DC removal, IQ imbalance compensation and optical hybrid IQ orthogonalization using the Gram-Schmidt orthogonalization procedure [16]. Next, the signal was resampled to 2 samples per symbol and then passed through the overlap-and-save frequency domain CD compensation and laser frequency offset compensation based on the FFT of the signal at the 4th power. Matched filtering was performed in the frequency domain using the same pulse-shaping filter used at the transmitter. Sampling frequency-offset compensation and timing recovery were carried out using a non-data-aided feed-forward symbol-timing estimator [23]. Next, synchronization is performed in order to facilitate data...
aided modulation transparent equalization by conventional correlate and delay algorithms (i.e. Schmidl-Cox) [24]. A training-symbol-aided decision directed least radius distance (TS-DD-LRD) [25] based fractionally spaced linear equalizer with 15 taps was used for fast convergence of the coefficients. The carrier phase was recovered using the superscalar parallelization based phase locked loop (PLL) combined with a maximum likelihood phase estimation [26]. Next, the fiber nonlinearity is compensated using either our novel adaptive nonlinear equalizer or perturbation based post nonlinearity compensation [11] (for comparison purposes). Finally, the symbols were mapped to bits and the bit error rate (BER) was counted over 100,000 bits and a soft-decision forward error correction (20% overhead) BER threshold of $2 \times 10^{-2}$ was considered.

Fig. 4. Receiver DSP

5. Discussion and results

In this section, we investigate the performance of proposed nonlinear equalizer against perturbation based nonlinear compensation at the receiver. All DSP blocks and parameters are identical for all schemes except for the nonlinearity compensation scheme. For all measurements, a root raised cosine (RRC) filter with a roll-off factor equal to 0.01 is chosen as a pulse-shaping filter for 32-GBaud dual-polarization (DP) 16QAM transmissions. Also $N_1$ and $N_2$ are equal to 5 and 10 respectively and consequently, there are $25 = (N_1 + 2N_2)$ adaptive coefficients for nonlinear equalizer. We have investigated the performance of the perturbation based NLC using two different implementations: 1) without quantization of coefficients and 2) with uniform quantization of perturbation coefficients into 15 coefficients.

In the experiment, the performance is investigated at 1 dBm launch power after 2560 km. Figure 5 summarizes the BER versus nonlinear equalizer memory depth curves for 32 GBaud DP-16QAM. The equalizer memory, $n_{\text{eff}}$, is normalized by maximum CD-induced pulse broadening, $n_{\text{max}}$. As shown in both figures, a negligible penalty is observed when $n_{\text{eff}} = n_{\text{max}}/2$ and by further decreasing the memory depth the performance decreases significantly. This justifies our motivation for Eq. (23). In addition, it should be noted that in the case that $n_{\text{max}}$ is unknown at the receiver the equalizer memory can be set manually by starting from a small $n_{\text{eff}}$ and gradually increasing the value until the desired performance is reached.
Next, we investigated the BER under different launch powers. The investigated distance is 2560 km. As shown in Fig. 6, when the power launched into the fiber is low, transmission is mainly limited by linear impairments and the BER is approximately the same for all algorithms. However, as the launch power increases, fiber nonlinearities become more significant and nonlinear compensation enables a lower BER than the conventional single carrier signal. The improvement is particularly significant when the launch power is larger than 1 dBm. As demonstrated in Fig. 6, performance of the proposed algorithm is better than (i) the perturbation based NLC with 25 uniform quantized coefficients, and (ii) comparable to more computationally complex perturbation based NLCs without coefficient quantization.

Next, we compare the achievable transmission distance for different launch powers with a forward error correction (FEC) pre-set BER threshold of $2 \times 10^{-2}$. The results are summarized in Fig. 7. In accordance with the results in Fig. 6, the achievable transmission distances of conventional single carrier transmission is significantly smaller without nonlinearity compensation. If we investigate the maximum transmission distance of all the systems at their respective optimum launch powers, transmission reach increases from 2365 km for only linear compensation to 2818, 2726 and 2904 km for adaptive nonlinear equalizer and perturbation based NLCs with and without coefficients quantization, respectively. In addition, the optimum launch power increases by 1 dB with fiber nonlinearity compensation.
Fig. 7. Experimental maximum transmission distance versus launch power for 32 GBaud SC-DP-16QAM at soft FEC BER threshold of $2 \times 10^{-3}$.

Finally, Fig. 8 shows convergence of the adaptive nonlinear equalizer over the time. Equations (7) and (8) can be used to initialize the equalizer. Here, we used zero for the initial adaptive nonlinear coefficient values. We used a large step size at the beginning of the training sequence to achieve fast convergence. After 1000 and 2000 symbol equalization, the step size was divided by 2 and algorithm switched to decision directed mode. In contrast to previously proposed nonlinear equalizers [11], grouping multiple nonlinear triplets removed the requirement to use DD-LMS with multiple iterations per symbol and suboptimal convergence condition in order to increase the convergence rate and equalizer stability.

Fig. 8. Convergence of adaptive perturbation coefficients over time.

6. Conclusion

We propose and experimentally demonstrate a novel low-complexity nonlinear equalizer. We achieved a transmission distance of 2818 km for a 32-GBaud DP-16QAM system. The proposed equalizer performance is comparable to the perturbation based nonlinearity compensation and previously studied nonlinear equalization methods. Unlike digital backpropagation, the proposed equalizer operates at one sample per symbol and requires only one computation step. In addition, it allows for compensation of nonlinear and linear impairments independently. In comparison to perturbation based nonlinearity compensation, our nonlinear equalizer does not require prior calculation of perturbation coefficients, symmetric dispersion maps or a large memory to store all possible perturbation coefficients for reconfigurable network scenarios. In contrast to previously proposed nonlinear equalizers, our algorithm takes advantage of common symmetries of the perturbation coefficients and avoids replication of operations. In addition, it uses only a few adaptive coefficients by grouping multiple nonlinear terms and dedicating only one coefficient to each group. Finally, its computational complexity is smaller than previously proposed adaptive nonlinear equalization techniques by more than one order of magnitude.