Digital compensation of cross-phase modulation distortions using perturbation technique for dispersion-managed fiber-optic systems

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Abstract: A digital compensation scheme based on a perturbation theory for mitigation of cross-phase modulation (XPM) distortions is developed for dispersion-managed fiber-optic communication systems. It is a receiver-side scheme that uses a hard-decision unit to estimate data for the calculation of XPM fields using the perturbation technique. The intrachannel nonlinear distortions are removed by intra-channel digital backward propagation (DBP) based on split-step Fourier scheme before the hard-decision unit. The perturbation technique is shown to be effective in mitigating XPM distortions. However, wrong estimations in the hard-decision unit result in performance degradation. A hard-decision correction method is proposed to correct the wrong estimations. Numerical simulations show that the hybrid compensation scheme with DBP for dispersion and intra-channel nonlinear impairments compensation and the perturbation technique for XPM compensation brings up to 3.7 dBQ and 1.7 dBQ improvements as compared with the schemes of linear compensation only and intra-channel DBP, respectively. The perturbation technique for XPM compensation requires only one-stage (or two-stage when hard-decision correction is applied) compensation and symbol-rate signal processing.

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References and links

1. Introduction

The performance of fiber-optic transmission systems is mainly limited by intra-channel [1–3] and inter-channel [4–7] nonlinear impairments. Digital/optical back propagation [8–11] based on split-step Fourier scheme (SSFS) is an effective method to compensate for intra-channel nonlinear impairments. Digital back propagation (DBP) requires computations of multiple fast Fourier transforms (FFTs) and two or more samples per symbol. Alternatively, perturbation techniques can be implemented either at the transmitter [12] or at the receiver [13,14] for the compensation of the intra-channel nonlinear impairments. For quasi-linear systems, the fiber-optic channel is considered to be linear to the leading order. The nonlinear interaction among signal pulses leads to distortions in the first and higher orders, which can be calculated using the perturbation theory. In transmitter side perturbation theory based compensation technique, the signal field is distorted by subtracting the first order field from the signal field at the transmitter. The fiber-optic system generates the first order field which cancels the distortion introduced at the transmitter. However, if the first order field is large, nonlinear interaction between this field and signal field causes additional distortions. In order to calculate the first order field, double summations have to be performed and the number of terms in the double summation grows quickly with accumulated dispersion. An interesting fact is that by employing symmetric electronic dispersion compensation (EDC), complex multiplications in the double summation become real multiplications which leads to significant reduction in computational complexity [15,16].

In a wavelength division multiplexing (WDM) system, a signal pulse in the given symbol slot interacts nonlinearly not only with the neighboring symbols of the same channel, but also with the symbols of the other channels due to cross phase modulation (XPM). The distortion due to XPM can be compensated for using the DBP [17], in which coupled nonlinear Schrödinger equations (NLSes) are solved using SSFS. This scheme is quite effective;
however, it requires huge computational resources since the step size should be about 3 km. The step size can be increased by factorizing the walk-off effect [18], however, the computational cost is still large. In [7, 19–21], inter-channel nonlinear distortion due to XPM is calculated using a perturbation theory. In this paper, we make use of the analytical expressions for the first order fields developed in [21] to compensate the distortion at the receiver. We first compensated for the intra-channel as well as inter-channel nonlinear distortions using the perturbation theory and the results showed that this compensation scheme brought only 0.5 dBQ improvement as compared to the case of linear compensation. Next, we investigated the possibility of compensating inter-channel distortions using the low-complexity intra-channel DBP (with a step size of 40 km) and inter-channel distortions using the perturbation technique. In this case, we found that the performance improvement is 2.4 dBQ as compared to the case of linear compensation for 2-channel WDM systems. Constellation diagram revealed the existence of small islands drifting away from the constellation points. These islands do not behave like amplified spontaneous emission (ASE) noise and they result from the wrong estimations in the hard-decision unit. Instead of choosing the closest constellation points for the received signal, we replaced the decision with the second-closest constellation point. Using the corrected hard-decision data, XPM distortion compensation is done again and now the performance improvement over the linear compensation scheme is 3.2 dBQ. We note that this method is applicable not only for WDM systems, but also for superchannel systems with multiple carriers [22]. The digital signal processing (DSP) technique for superchannel systems is discussed in [22–24]. For simplicity, we have ignored the polarization dependence of the signals and limited our study to single-polarization WDM systems.

2. Perturbation theory

2.1 Perturbation theory for Gaussian pulses

Optical signal distortions due to fiber nonlinearity in a fiber-optic link can be calculated using a perturbation technique [7, 19–21]. The evolution of the optical field envelope in a fiber-optic link is described by the NLSE, which can be written in the lossless form as

\[
j \frac{\partial u}{\partial z} = \frac{\beta_2(z)}{2} \frac{\partial^2 u}{\partial T^2} + \gamma(z) |u|^2 u = 0,
\]

where

\[
u(z,T) = \exp\left[ w(z) / 2 \right] q(z,T),
\]

\[
w(z) = \int_0^\infty \alpha(s) ds, \quad \gamma(z) = \gamma_0 \exp\left[ -w(z) \right],
\]

\(\alpha, \beta_2, \) and \(\gamma_0\) are the loss, dispersion and nonlinear coefficients, respectively. Consider two channels of a WDM system. The total field envelope is

\[
u = \nu_1 + \nu_2,
\]

where \(\nu_k\) is the field envelope of the \(k\)th channel, \(k = 1, 2\). Substituting Eq. (4) into Eq. (1) and ignoring the four wave mixing (FWM) terms, we obtain

\[
j \frac{\partial \nu_k}{\partial z} - \frac{\beta_2(z)}{2} \frac{\partial^2 \nu_k}{\partial T^2} = -\gamma(z) \left[ |\nu_k|^2 + 2 |\nu_l|^2 \right] \nu_k, \quad \text{for} \quad k = 1, 2 \text{ and } l = 3-k.
\]

Without loss of generality, we consider the interaction between a probe pulse (in symbol slot 0 of channel 1) and multiple pump pulses (in channel 2). We assume that the leading order
solution of Eq. (5) is linear and treat the nonlinear terms on the right-hand side as perturbations. For the case of Gaussian pulses, the input optical signals can be written as

\[
\begin{align*}
\text{Eq. (6)} & \quad u_i(0,T) = \sqrt{P} a_i g(0,T), \\
\text{Eq. (7)} & \quad u_z(0,T) = \sqrt{P} \sum_{n=-N_{\text{sym}}}^{N_{\text{sym}}} b_n g(0,T-nT_s) \exp(-j\Omega t), \\
\text{Eq. (8)} & \quad g(0,T) = \exp \left( -\frac{T^2}{2T_0^2} \right),
\end{align*}
\]

where \( P \) is the power, \( a_0 \) and \( b_n \) are the random data, \( (2N_{\text{sym}} + 1) \) is the total number of symbols, \( T_s \) is the symbol interval, \( \Omega \) is the channel separation in radians, \( T_0 \) is the half-width at 1/e-intensity point of the Gaussian pulse. Using the perturbation technique, we take \( \gamma_0 \) as a small parameter and expand the field in channel \( k \) into a series

\[
\begin{align*}
\text{Eq. (9)} & \quad u_k = u_k^{(0)} + \gamma_0 u_k^{(1)} + \gamma_0^2 u_k^{(2)} + \ldots, \quad k = 1, 2,
\end{align*}
\]

where \( u_k^{(m)} \) denotes the \( m \)-th-order solution. For Gaussian pulses, the linear solutions \( u_k^{(0)} \) have closed form expressions. Substituting Eq. (9) into Eq. (5) and collecting all the terms that are proportional to \( \gamma_0 \), we find the governing equation for the first-order solution as

\[
\begin{align*}
\text{Eq. (10)} & \quad j \frac{\partial u_k^{(0)}}{\partial z} - \frac{\beta(z)}{2} \frac{\partial^2 u_k^{(0)}}{\partial T^2} = -e^{-w(z)} \left[ |u_k^{(0)}|^2 + 2|u_k^{(0)}|^2 \right] u_k^{(0)}, \quad k = 1, 2 \quad \text{and} \quad l = 3-k.
\end{align*}
\]

The first and second terms on the right hand side of Eq. (10) account for intra-channel and XPM effects, respectively. Substituting the linear solutions into Eq. (10), we find the XPM distortion on the pulse of channel 1 as \([21]\)

\[
\begin{align*}
\text{Eq. (11)} & \quad \Delta u_1^{\text{XPM}}(T) = \gamma_0 u_1^{(1),\text{XPM}} = 2j\gamma_0 P^{3/2} a_0 \sum_{m=-N_{\text{sym}}}^{N_{\text{sym}}} b_m h_m X_{mn}(L_{\text{tot}}, T),
\end{align*}
\]

where \( L_{\text{tot}} \) is the total transmission distance, and

\[
\begin{align*}
\text{Eq. (12)} & \quad X_{mn}(L_{\text{tot}}, T) = \int_0^{L_{\text{tot}}} \exp \left\{ \frac{-\eta(s)}{\sqrt{\delta(L_{\text{tot}}, s)}} \right\} \exp \left( \frac{(D + jT)^2}{\delta(L_{\text{tot}}, s)} \right) ds, \\
\text{Eq. (13)} & \quad \eta(z) = -\frac{T_0^2}{T_i(z)} \left( \frac{\alpha(z)}{\beta(z)} \right)^2, \quad \eta'(z) = \eta(z) \exp \left( -\sum_{i=1}^{3} C_i R_i + \frac{\alpha(z)}{R} \right), \\
\text{Eq. (14)} & \quad \delta(z,s) = 1/R - j2 \left[ S(z) - S(s) \right], \quad S(z) = \int_0^z \frac{\beta(s) ds}{\beta(z)}, \\
\text{Eq. (15)} & \quad C_i(z) = mT_i + S(z)\Omega, \quad C_2(z) = nT_i + S(z)\Omega, \quad C_3(z) = 0, \\
\text{Eq. (16)} & \quad R_i = R_s = 1/2T_i, \quad R_c = \frac{1}{2(\eta(z))^2}, \quad T_i = \sqrt{T_0^2 - JS(z)}, \\
\text{Eq. (17)} & \quad R = R_i + R_s + R_c, \quad C = C_i R_i + C_2 R_s + C_3 R_c, \quad D = jC / R.
\end{align*}
\]

From Eq. (10), we find that the forcing functions of intra-channel and XPM effects in channel 1 are \( -e^{-w(z)} |u_k^{(0)}|^2 u_k^{(0)} \) and \( -2e^{-w(z)} |u_k^{(0)}|^2 u_k^{(0)} \), respectively. Using the symmetry, we obtain
the intra-channel distortion simply by setting the channel separation $\Omega = 0$ and removing the XPM factor 2 in Eq. (11), which is

$$
\Delta u^{(1),\text{intra}}_t (T) = \frac{\gamma_0}{2} \sum_{m=-N_{\text{sym}}}^{N_{\text{sym}}} \sum_{n=-N_{\text{sym}}}^{N_{\text{sym}}} a_m a_n^* X_m (L_{\text{tot}}, T),
$$

where $a_m$ and $a_n$ are the random data in channel 1. This expression of intra-channel distortion includes self-phase modulation (SPM), intra-channel XPM (IXPM) and intra-channel FWM (IFWM) effects.

### 2.2 Perturbation theory for non-Gaussian pulses

For non-Gaussian pulses, optical signal distortions due to fiber nonlinearity cannot be calculated analytically. It is shown that a summation of time-shifted Gaussian pulses can be used to fit a non-Gaussian pulse, which makes it possible to derive analytical expressions for nonlinear distortions [21]. A similar approach has been used in quantum chemistry [25]. A non-Gaussian pulse $h(T)$ is approximated by

$$
h' (T) = \sum_{k=1}^{K} \xi_k \exp \left[ -\frac{(T - \mu_k T'_t)^2}{2(\theta_k T'_t)^2} \right]
$$

where $\xi_k$, $\mu_k$ and $\theta_k$ are fitting parameters, $K$ is the number of time-shifted Gaussian functions. The fitting parameters are optimized using the least squares method (LSM). Using Eq. (19), the nonlinear distortions can be derived following a similar procedure as in Section 2.1.

$$
\Delta u^{(1),\text{intra,NG}}_t (T) = \frac{\gamma_0}{2} \sum_{m=-N_{\text{sym}}}^{N_{\text{sym}}} \sum_{n=-N_{\text{sym}}}^{N_{\text{sym}}} a_m a_n^* Y_m (L_{\text{tot}}, T),
$$

$$
\Delta u^{\text{XPM,NG}}_t (T) = \frac{\gamma_0}{2} \sum_{m=-N_{\text{sym}}}^{N_{\text{sym}}} \sum_{n=-N_{\text{sym}}}^{N_{\text{sym}}} b_m b_n^* Y_m (L_{\text{tot}}, T),
$$

where the parameters are the same as those defined in Section 2.1, except for the following ones:

$$
T_{k,\ell} = \sqrt{(\theta_k T'_t)^2 - jS(z)}, \quad \eta(z) = -e^{-w(z)} \xi_k \xi_{k+1} \xi_{k+2} \xi_{k+3} \frac{\theta_k \theta_{k+1} \theta_{k+2} \theta_{k+3}}{T_{1,1} T_{1,2} T_{1,3}},
$$

$$
C_1(z) = \tau_{w, k}, \quad C_2(z) = \tau_{w, k+1}, \quad C_3(z) = \mu_k T'_t,
$$

$$
R_1(z) = \frac{1}{2T_{1,2}^2}, \quad R_2(z) = \frac{1}{2(T_{1,2} T_{1,3})^2}, \quad R_3(z) = \frac{1}{2T_{1,3}^2},
$$

$$
\tau_{w, \ell} = (n + \mu_k) T'_t + S(z) \Omega.
$$

Equations (11) or (21) correspond to the case when there is a single pulse in channel 1 (at symbol slot 0). When there are multiple pulses, the nonlinear interaction between a neighboring pulse of channel 1 (e.g. pulse at symbol slot 1) and the multiple pulses of channel 2 leads to distortion on the pulse at the symbol slot 0 of channel 1. This is because the neighboring pulses as well as XPM fields broaden due to dispersion and walk-off effects such
that they could appear at symbol slot 0 at various propagation distances, leading to additional XPM distortions. Therefore, Eq. (21) is modified as

$$\Delta u_{\text{XPM,NG}}(T) = 2j \gamma_0 P^{3/2} \sum_{l=1}^{M} \sum_{n=-N_{\text{sym}}}^{N_{\text{sym}}} \sum_{m=-N_{\text{sym}}}^{N_{\text{sym}}} a_i b^*_{\text{sym}} Y^{(i)}_{\text{sym}}(L_{\text{tot}}, T),$$  

(27)

where \(a_i\) is the random data on channel 1, \(Y^{(i)}_{\text{sym}}\) is the matrix which corresponds to the nonlinear interaction between the pulse at the symbol slot \(l\) of channel 1 and multiple pulses of channel 2. \(M\) is the number of neighboring symbols up to which the nonlinear distortion is significant. Similarly, for intra-channel distortions, Eq. (20) is modified as

$$\Delta u_{\text{intra,NG}}(T) = j \gamma_0 P^{3/2} \sum_{l=1}^{M} \sum_{m=-N_{\text{sym}}}^{N_{\text{sym}}} \sum_{n=-N_{\text{sym}}}^{N_{\text{sym}}} a_i a^*_{\text{intra}} Y^{(i)}_{\text{intra}}(L_{\text{tot}}, T).$$  

(28)

So far we considered the XPM distortion due to a single neighboring channel. If there are \(N_{\text{sub}}\) neighboring channels, total nonlinear distortion due to XPM is

$$\Delta u_{\text{XPM}} = \sum_{k=1}^{N_{\text{sub}}} \Delta u_{\text{XPM,NG}},$$  

(29)

where \(\Delta u_{\text{XPM,NG}}\) represents the XPM distortion due to the neighboring channel \(k\) and is calculated using Eq. (27).

### 2.3 Nonlinearity compensation using perturbation technique

Using the perturbation technique, the distorted output signal of a fiber-optic link can be written as

$$u_{\text{out}} = u_{\text{in}} + \Delta u_{\text{intra}} + \Delta u_{\text{XPM}} + \Delta u_{\text{H}}$$  

(30)

where \(u_{\text{out}}\) and \(u_{\text{in}}\) are the input and output signals, respectively; \(\Delta u_{\text{intra}}\) and \(\Delta u_{\text{XPM}}\) are the first order intra-channel and XPM distortions calculated by Eqs. (27) and (29), respectively; \(\Delta u_{\text{H}}\) is the high-order component that is neglected by the first-order perturbation theory [19]. To the first-order accuracy, the fiber nonlinearity can be compensated by

$$u_{\text{comp}} = u_{\text{out}} - \Delta u_{\text{intra}} - \Delta u_{\text{XPM}}.$$  

(31)

Perturbation-based nonlinearity compensation can be implemented at the transmitter side and/or at the receiver side [12–14]. In the transmitter side perturbation scheme, the accurate input data \((a_{\text{intra}}, b_{\text{intra}})\) is available for the perturbation calculation of nonlinear distortions. However, due to the limitations of first-order theory, the high-order component \(\Delta u_{\text{H}}\) will co-propagate and interact with signals in the fiber-optic link which will introduce additional distortions. In the receiver side perturbation scheme, the high-order component does not propagate; but the input data is not available so that perturbation calculation has to be done based on estimated data from the distorted signals. In single channel systems, transmitter side and receiver side perturbation schemes have almost the same performance [13].

### 3. Results and discussions

#### 3.1 Compensation of both intra-channel and XPM distortions using perturbation technique

As the first case, we investigated a receiver side compensation scheme that mitigates both intra-channel and XPM distortions using the perturbation technique. To reduce computational complexity (i.e., the value of \(M\) and the size of the matrix \(Y_{\text{sym}}\)), we consider a dispersion-managed (DM) fiber-optic system, as shown in Fig. 1. Unless otherwise specified, the system configuration is as follows: symbol rate per channel = 28 Gbaud, modulation = 16-quadrature
amplitude modulation (16-QAM), channel spacing = 50 GHz, Tx and local oscillator (LO) laser linewidth = 100 kHz, amplifier spacing = 80 km, number of fiber spans = 20, number of symbols simulated = 32768 per channel. The dispersion, loss, and nonlinear coefficients of the transmission fiber are $D_{TF} = 16.5 \text{ ps/nm/km}$, $\alpha_{TF} = 0.2 \text{ dB/km}$, and $\gamma_{TF} = 1.1 \text{ W}^{-1}\text{km}^{-1}$, respectively. For dispersion compensating fiber (DCF), $D_{DCF} = -117.7 \text{ ps/nm/km}$, $\alpha_{DCF} = 0.5 \text{ dB/km}$, and $\gamma_{DCF} = 4.4 \text{ W}^{-1}\text{km}^{-1}$. Gain of the first and second stages of the amplifiers are $G_1 = 13.0 \text{ dB}$ and $G_2 = 8.4 \text{ dB}$, respectively. The residual dispersion per span ($= D_{TF}L_{TF} + D_{DCF}L_{DCF}$) is 50 ps/nm. The standard split-step Fourier scheme (SSFS) is used to simulate the signal propagation in the fiber-optic link. The computational bandwidths are 0.22 THz and 0.45 THz for the cases of 2-channel and 5-channel WDM systems, respectively; and the maximum nonlinear phase shift per step is 0.0005 radians. A Nyquist pulse with a roll-off factor $a = 0.6$ is used, defined as

$$x(t) = \sin \left( \frac{t}{T_s} \right) \cos (a \pi t / T_s) \left[ 1 - (2at / T_s)^2 \right].$$  \hspace{1cm} (32)$$

Six time-shifted Gaussian pulses are used to fit the Nyquist pulse, with the parameters optimized by the least squares method (LSM) given in Table1. [21]. As the roll-off factor decreases, the number of pulses required increases. However, this does not increase the computational complexity of the compensation scheme, as the calculation of $Y_{m\alpha}$ matrixes are done off-line and stored as look-up tables.

<table>
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<th>$k$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
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<tr>
<td>$\xi_k$</td>
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<td>0.5342</td>
<td>-0.2588</td>
<td>-0.2588</td>
<td>0.0289</td>
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</tr>
<tr>
<td>$\mu_k$</td>
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<td>-0.0042</td>
<td>0.9884</td>
<td>-0.9884</td>
<td>1.8314</td>
<td>-1.8314</td>
</tr>
<tr>
<td>$\theta_k$</td>
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<td>0.5909</td>
<td>0.4932</td>
<td>0.4932</td>
<td>0.3781</td>
<td>0.3781</td>
</tr>
</tbody>
</table>

At the output of the fiber-optic link, a demultiplexer (DMUX) separates signals into different channels. Second order Gaussian band pass filters (BPFs) centered at each channel with 3-dB bandwidth of 42 GHz are used as demultiplexing filters. The output signals from the coherent receivers are then converted into the digital domain using analog-to-digital convertors (ADCs). In the receiver digital signal processing (DSP) block, two samples per symbol are used for the compensation of residual dispersion. The output of dispersion...
compensator (DC) is down-sampled to one sample per symbol. After that, carrier phase recovery (CPR) is implemented using the feed-forward method [26]. The output signal of CPR is used for hard decision which uses a threshold device to make decisions based on the proximity of the signal to the complex amplitude levels of 16-QAM. The rectangular regions about the constellation points are used for hard decision. Perturbation calculations for both intra-channel and XPM distortions are carried out using the hard-decision data, which approximate the input data sequences \( \{a_n\} \) and \( \{b_n\} \) required for perturbation calculation in Eqs. (27) and (28). The coefficient matrix \( Y_{mn} \) are stored in a lookup table. When the channel separations between the probe and the pump channel are 50 GHz, 100 GHz, 150 GHz, and 200 GHz, the numbers of neighboring symbols included (i.e., \( M \) in Eq. (27)) are 2, 1, 1, and 0, respectively. Then intra-channel and XPM distortions are compensated using Eq. (31) and the output of CPR is used as \( u_{out} \).

For simplicity, we first consider a 2-channel WDM system. Figures 2(a) and 2(b) show the constellations of recovered signals by dispersion compensation only and by nonlinear compensation of both intra-channel and XPM distortions using the perturbation technique, respectively. We note that in simulations the signal constellation becomes enlarged after nonlinearity compensation. Before calculating bit error ratio (BER), the power of the compensated signal is normalized to that of the input signal. The BER is calculated by error counting. Figure 3 shows the Q-factor as a function of average launch power per channel \( P_{ave} \). The Q-factor is converted from the BER using \( Q = \sqrt{2\text{erfc}^{-1}(2\times\text{BER})} \) and \( Q(dB) = 20\log_{10}Q \). Perturbation-based nonlinearity compensation brings 0.5 dBQ improvement. The relatively smaller improvement is mostly due to the inaccurate data after the hard decision.
3.2 Nonlinearity compensation using DBP for intra-channel impairments compensation and the perturbation technique for XPM compensation

In this section, we choose to compensate for intra-channel nonlinearities using intra-channel DBP based on SSFS [8], since the intra-channel DBP has high accuracy and reasonable computational cost. However, for XPM compensation, the computational cost of inter-channel DBP is very large. The required step size is as small as a few kilometers [17]. Perturbation-based XPM compensation requires much less computational cost, since it only requires one-stage compensation and symbol-rate signal processing. Therefore, we investigated a hybrid nonlinearity compensation scheme, which compensates for dispersion and intra-channel nonlinearities using intra-channel DBP and compensates for XPM distortions using the perturbation technique. The scheme diagram is shown in Fig. 4. The ADC outputs with two samples per symbol are launched into the intra-channel DBP units to compensate for intra-channel nonlinear distortions. After that, second order Gaussian low pass filters (LPFs) with 3-dB bandwidth of 19 GHz are used as noise limiting filters. Then, CPR and hard decision are implemented using one sample per symbol. XPM perturbation calculation and compensation are carried out based on the hard-decision data.

Fig. 4. Diagram of a hybrid nonlinearity compensation scheme using DBP for intra-channel impairments compensation and the perturbation technique for XPM compensation. LPF: low pass filter.

As intra-channel DBP provides more accurate compensation of intra-channel nonlinear distortions than the perturbation technique, the hard-decision unit provides more accurate estimation of the input data. As a result, the XPM perturbation calculation and compensation becomes more accurate than the scheme in Section 3.1. Figure 5 shows the constellations of recovered signals. Comparing Fig. 5(c) with Fig. 5(b), we see that the perturbation technique is effective to compensate XPM distortions. However, the improvement in Q-factor by XPM compensation using the perturbation technique is still relatively smaller [see Fig. 8]. The main reason is that the presence of XPM distortions results in wrong estimations in the hard-decision unit. From Fig. 5(c), we see that in addition to 16 constellation points (enlarged due to ASE noise), there exist small islands drifting away from the 16 constellation points. The drifting islands do not behave as ASE noise and we found that they result from the wrong estimations in the hard-decision unit. We employed a hard-decision correction method [as shown in Fig. 6] to correct the wrong decisions and re-compensated XPM distortions using the perturbation technique based on the corrected data. The drifting islands were mostly removed by the hard-decision correction method, as shown in Fig. 5(d).
Fig. 5. Constellations of recovered signals in a 2-channel WDM system: (a) linear compensation only; (b) nonlinearity compensation using intra-channel DBP only (step size = 40 km); (c) nonlinearity compensation using intra-channel DBP and the perturbation technique for XPM; (d) after hard-decision correction. (average power per channel $P_{ave} = -3$ dBm)

Fig. 6. Diagram of a nonlinearity compensation scheme using hard-decision correction.

Figure 6 shows the hard-decision correction method. The first step is to locate the wrong decisions. This is realized by finding the drifting islands in the constellation [Fig. 5(c)] of the output signals of the XPM compensation unit. We assume that a wrong decision occurred if the distance between an output signal point and its closest constellation point is larger than a certain threshold distance, that is

$$|\text{sig}_{\text{out}} - \text{sig}_{\text{constel}}| > r$$  \hspace{1cm} (33)

where $\text{sig}_{\text{out}}$ and $\text{sig}_{\text{constel}}$ are the normalized output signal and its closest constellation point, respectively; $r$ is the threshold distance. The circular regions about constellation points are used for hard-decision correction. Then we trace back to the hard-decision unit and replace the decision with the second-closest constellation point (rather than the closest one) of that CPR output signal point. Using the corrected hard-decision data, the perturbation calculation and compensation for XPM distortions are implemented for a second time. In the second round XPM compensation, corrected data from hard-decision correction is used to re-calculate the XPM distortion ($\Delta u_{\text{XPM}}$) using Eqs. (27) and (29), i.e., the input data sequence $\{a_n\}$ and $\{b_n\}$ appearing in Eqs. (27) and (29) are approximated by the corrected data from the hard-decision correction unit. Then, the XPM distortion is removed using $u_{\text{comp}} = u_{\text{out}} - \Delta u_{\text{XPM}}$, where the CPR output is used as $u_{\text{out}}$. In the hard-decision correction unit, both the CPR output ($u_{\text{out}}$) and the decisions of first-stage XPM compensation are used as inputs to find the corrected data. The matrix $Y_{mn}$ calculated before is re-used in the second-stage
compensation. In simulations, the optimal threshold distances are obtained by sweeping the range \([0 1]\). The distance between the nearest constellation points is normalized to 2. Figure 7 shows the Q-factor as a function of the normalized threshold distance. It shows that the optimum threshold distance is dependent on launch power and system configuration. In practical systems, the optimal threshold values can be pre-determined by numerical simulations and then built into the DSP for transmission systems.

![Fig. 7. Q-factor versus normalized threshold distance.](image)

Figure 8 compares the Q-factors of different compensation schemes. The Q improvement is defined as the difference between maximum Q-factors of different schemes. Figure 8(a) shows the case when the step size of intra-channel DBP is 2 km. Intra-channel DBP brings 2.0 dBQ improvement as compared with linear compensation only. Using the perturbation technique to compensate for XPM distortions, an additional 0.7 dBQ improvement is obtained. Using the hard-decision correction method, the improvements are 3.7 dBQ and 1.7 dBQ as compared with linear compensation and intra-channel DBP, respectively. We also investigated the case when a large step size is used for intra-channel DBP. Figure 8(b) shows slightly performance degradation when the step size is 40 km. The scheme with hard-decision correction shows 3.2 dBQ and 1.4 dBQ improvements as compared with linear compensation and intra-channel DBP, respectively. When the correct symbols are artificially used (instead of the output of the hard-decision correction unit), the BER becomes very small \(<10^{-6}\). This implies that the first-order perturbation based compensation technique is able to effectively remove the XPM distortions and the performance degradation is due to hard-decision error. This can be seen by the few islands in Fig. 5(d) even after hard-decision correction.

![Fig. 8. Q-factor versus average launch power per channel in a 2-channel WDM system: (a) intra-channel DBP step size = 2 km; (b) intra-channel DBP step size = 40 km.](image)

For a more general case, we investigated a 5-channel WDM system with the channel spacing of 50 GHz. Figure 9 shows the Q-factors of different schemes. In this case, the perturbation technique for XPM leads only to a slight performance improvement. However, with hard-decision correction, the additional improvements are 1.2 dBQ and 1.0 dBQ as compared to intra-channel DBP only, for the cases of 2 km and 40 km step sizes, respectively.
Total improvements as compared to the case of linear compensation are 2.5 dBQ and 2.3 dBQ for the cases of 2 km and 40 km step sizes, respectively.

Fig. 9. Q-factor versus average launch power per channel in a 5-channel WDM system: (a) intra-channel DBP step size = 2 km; (b) intra-channel DBP step size = 40 km.

4. Conclusions

We have investigated a digital compensation scheme based on a perturbation theory to compensate for fiber nonlinearities in dispersion-managed fiber-optic systems. The scheme uses the data obtained from a hard-decision unit at the receiver to do perturbation calculation. In the scheme that uses perturbation technique to compensate for both intra-channel and XPM distortions, the performance improvement is small due to the inaccurate data after the hard-decision unit and the limitations of the first-order theory. We then considered a hybrid scheme, where the intra-channel distortions are removed by intra-channel DBP and the XPM distortions are compensated by the perturbation technique. Better performance improvement is obtained since intra-channel DBP provides more accurate compensation for intra-channel distortions which results in more accurate data in the hard-decision process. We also studied a hard-decision correction method to correct the wrong estimations in the hard-decision unit. We located the wrong estimated signals from the constellation of the recovered signal and then traced back to replace the hard-decision data of those wrong estimations with the second-closest constellation point (rather than the closest one). Numerical simulations show that the hybrid scheme brings up to 3.7 dBQ and 1.7 dBQ improvements as compared with the schemes of linear compensation only and intra-channel DBP, respectively. The perturbation technique for XPM compensation requires only one-stage (or two-stage when hard-decision correction is applied) compensation and symbol-rate signal processing. The results are also applicable to superchannel systems with multiple carriers.