Dispersion-enhanced phase noise effects on reduced-guard-interval CO-OFDM transmission

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Abstract: Unlike conventional CO-OFDM systems, we show in this paper that reduced-guard-interval (RGI) CO-OFDM systems experience subcarrier-dependent phase noise (PN) from the local oscillator laser. This phenomenon manifests in RGI-CO-COFM systems because the chromatic dispersion (CD) induced walk-off becomes comparable to the OFDM symbol length. We term this phenomenon the dispersion enhanced PN (DEPN). In this work an analytical study of the impact of DEPN on CO-OFDM transmission is conducted. We develop a system-level analytical model and calculate the variance of the dispersion-induced subcarrier-dependent phase rotation term (PRT) using two different distribution patterns of pilot subcarriers (PS). Moreover, we present a bit error rate (BER) estimator to quantify the system performance degradation due to PRT. Numerical simulations are then performed to verify the analytical model. Finally, we propose a grouped maximum-likelihood (GML) phase estimation approach to mitigate the DEPN impairment, and demonstrate a 0.7-1.7 dB SNR improvement at BER = 10−3 for typical 100 Gb/s RGI CO-OFDM systems.

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References and links

1. Introduction

Coherent optical OFDM (CO-OFDM) systems have been actively investigated as a promising candidate for Ethernet transport at 100 Gb/s and beyond [1–3]. Because of the large chromatic dispersion (CD) in long haul optical transmission, a large cyclic prefix (CP) for CO-OFDM is usually required to combat inter-symbol interference (ISI). However, this will induce a large overhead, unless transmitting OFDM signals with much longer symbol durations, which reduces the ratio of CP length to symbol duration. Normally, more than one thousand subcarriers are required to keep a reasonably small CP overhead [2]. However, a large number of subcarriers tends to produce a high peak to average power ratio (PAPR), reducing the system tolerance to fiber nonlinearities. Moreover, CO-OFDM signals with long symbol durations suffer from a large inter-carrier interference (ICI) from the laser phase noise (PN). Therefore, considering these disadvantages, conventional CO-OFDM systems become less attractive than single carrier systems [4]. Recently, a reduced-guard-interval (RGI) CO-OFDM system was proposed to realize 100 Gb/s and beyond transmission [5–10]. In this novel scheme, CD and polarization mode dispersion (PMD) are compensated before OFDM demodulation [10], which enables the use of a smaller number of subcarriers and achieves a smaller CP overhead than that of the conventional CO-OFDM. Although the computation complexity of RGI CO-OFDM in terms of dispersion compensation becomes larger compared to conventional CO-OFDM, it becomes nearly the same as that of single carrier systems with two-stage dispersion compensation [11]. More importantly, RGI CO-OFDM is more tolerant to fiber nonlinearity and laser PN, compared with conventional CO-OFDM [9], while preserving its advantages such as high spectral efficiency.

The interplay between dispersion and laser PN in dispersion uncompensated transmission has been extensively studied for single carrier systems [12–14] and direct-detection optical OFDM systems [15, 16]. This effect is negligible in conventional CO-OFDM systems, because the symbol duration is much longer than the dispersion induced walk-off and thus the laser PN is highly correlated across all the subcarriers. Therefore, the phase rotations of all the subcarriers are almost the same and can be fully compensated by sending pilot subcarriers (PS’s) [17]. However, for a RGI CO-OFDM signal, the symbol duration is often smaller than the dispersion length after transmission over thousands of kilometers. Therefore, different subcarriers will experience different phase rotations because of local oscillator laser PN and the relatively large walk-off between subcarriers. This unique phenomenon in RGI CO-OFDM systems will induce an additional OSNR penalty if the phase rotations across all the subcarriers are still compensated as “common”. 
In this paper, this dispersion-enhanced PN (DEPN) in RGI CO-OFDM is studied using both analytical and Monte-Carlo (MC) approaches. We analytically calculate the variance of the phase rotation term (PRT) for each subcarrier when the pilot subcarriers (PS) are uniformly distributed or centered over the signal spectrum. We show that the dispersion-induced average PRT variance is less for uniformly distributed PS’s (UD-PS’s) than centered PS’s (C-PS’s), and subsequently UD-PS’s provides a better BER performance than C-PS’s. In addition, an approximate BER estimator is presented, assuming the ICI is Gaussian distributed. Furthermore, OFDM signals with various numbers of subcarriers are simulated and their system performances are compared. We find that CO-OFDM with a smaller number of subcarriers experience less ICI but more PRT. Therefore, there is an optimal number of subcarriers, which is shown to be 80 for single polarization 56 Gb/s transmission over distances less than 4800 km, assuming the oversampling factor is 1.6. Finally, we propose a PRT compensator to mitigate the DEPN. The compensator employs a grouped maximum-likelihood phase estimation and 0.7-1.7 dB OSNR improvement at a BER = 10^{-3} is achieved.

2. System Model

![Block diagram of RGI CO-OFDM systems.](Fig. 1)

Figure 1 depicts the schematic setup of polarization-multiplexed RGI CO-OFDM transmissions. The CD is compensated before OFDM demodulation using an overlap frequency domain equalizer [18] and the dynamic fiber variations such as PMD can be compensated using the conventional one-tap OFDM channel estimation [19]. This configuration enables the use of a small FFT-size to generate OFDM signals, because the CP overhead is no longer constrained by the large amount of CD in long haul optical transmission.

![The communication channel model.](Fig. 2)

For our theoretical study in this paper, we consider a typical single polarization RGI CO-OFDM system, and its communication channel model is shown in Fig. 2. The carrier frequency offset, fiber nonlinearities and polarization effects are neglected for the sake of simplicity. The transmitted signal is the same as the conventional CO-OFDM, which can be expressed as

\[ s(t) = \sum_{k=1}^{N_c} c_k e^{j2\pi k \nu f_t} \]

where \( c_k \) is the symbol transmitted on the subcarrier \( k \), \( N_c \) is the number of the subcarriers and \( \Delta f \) is the subcarrier frequency spacing. In (1), the CP length is set to zero and the OFDM symbol index is omitted for simplicity.
Laser PN $\phi(t)$ is modeled as a Wiener process with a variance of $2\pi\beta t$, where $\beta$ is the laser linewidth [15]. The phase fluctuations will be added at both the transmitter and the receiver, by multiplying $\exp[j\phi(t)]$. Therefore the received signal $r(t)$ is given by

$$
 r(t) = \left\{ s(t) \cdot e^{j\phi(t)} \right\} \cdot e^{j\phi(t)} + z(t) 
$$

$$
 = \sum_{k=1}^{N} c_k \cdot e^{2\pi j D_k (n+D_k)} \cdot \delta(t-T_k) \cdot e^{j\phi(t)} + z(t) 
$$

$$
 = \sum_{k=1}^{N} c_k \cdot e^{2\pi j D_k (n+D_k)} \cdot \delta(n+D_k) + z(n) 
$$

where $\phi(t)$ is the PN caused by transmitter laser and LO laser, respectively, and $*$ denotes the convolution operator. $z(t)$ is the ASE noise, which is modeled as AWGN noise with a zero mean and a variance of $\sigma_{\text{ASE}}^2$. $h(t) = \delta(t-T_s)$ is the fiber channel response, which only contains CD in this model. $T_k$ represents the walk-off of fiber dispersion relative to the first subcarrier, and is given by $T_k = \left[ D/L \left( k - 1 \right) / \lambda v \right]$, of which $D$ is the dispersion parameter, $L$ is the fiber length, $\lambda$ is the carrier wavelength and $c$ is the speed of light in vacuum.

At the receiver, the received signal $r(t)$ is sampled and passed through the filter $h_n(n) = \delta(n+D_k)$ to reverse the effects of CD, where $D_k = T_k/T_s$ and $T_s$ is the sample duration. Therefore the output signal $r_{dc}(n)$ is given by

$$
 r_{dc}(n) = r(n) * h_n(n) 
$$

$$
 = \sum_{k=1}^{N} c_k \cdot e^{2\pi j D_k (n+D_k)} \cdot \delta(n+D_k) + z(n) 
$$

Now we can see that while the PN from the transmitter laser is common to all the subcarriers, the PN from the LO laser for each subcarrier is determined by the CD-induced walk-off $D_k$. Fig. 3(b) illustrates the dispersion-induced phase differences across the subcarriers at the receiver side, while there is no such phase difference at the transmitter as shown in Fig. 3(a).

![Fig. 3. (a) The applied laser PN at the transmitter side; (b) the applied laser PN at the receiver side.](image)

The variance of ICI is proportional to the symbol duration, and it doesn’t change during transmission because the frequency bandwidth for each subcarrier is very small. However, the phase rotation varies depending on the transmission length and symbol duration. The effect of the phase rotation difference between subcarriers is specified as PRT, which was first used to describe a similar phenomenon in direct-detection optical OFDM [15].
After FFT, we obtain the well-known expression for the received subcarriers \( R(k) \) \[20\]

\[
R(k) = c_k \cdot I_0(0) + ICI(k) + Z(k)
\]

where \( Z(k) \) is the DFT of sampled ASE noise \( z[n] \) and \( ICI(k) \) is given by

\[
ICI(k) = \sum_{i \neq k} c_i \cdot I^1(i - k)
\]

with \( I(p) = \frac{1}{N} \sum_{n=0}^{N-1} e^{j\frac{2 \pi m n}{N} \phi_k(n) + \phi_k(n+D_p)} \)

where \( N \) is the FFT size.

3. Performance analysis

3.1. Analytical results of PRT effects

It is known that PN effects on OFDM signals consist of power degradation, phase rotation and ICI \[20\]. In conventional CO-OFDM, the phase rotation is almost identical within one OFDM symbol due to the relatively small walk-off compared with the symbol duration. When sending a sufficiently large number of PS’s and averaging them to remove the noise interference, the phase rotation can be estimated and completely compensated. In addition, the distribution pattern of the PS’s makes no difference to the estimation and compensation. However, for RGI CO-OFDM, we find that not only the phase rotation is no longer identical among different subcarriers, but also the PRT variance differs as the PS distribution pattern changes. Here we study two common PS distribution patterns: 1) uniform distribution, denoted as UD-PS’s, and 2) centered, denoted as C-PS’s.

Using the small PN assumption, we have

\[
I_0(0) = \frac{1}{N} \sum_{n=0}^{N-1} e^{j\frac{2 \pi m n}{N} \phi(n) + \phi(n+D_p)} \approx 1 + j \left[ \frac{1}{N} \sum_{n=0}^{N-1} \phi(n) + \frac{1}{N} \sum_{n=0}^{N-1} \phi(n+D_p) \right]
\]

The imaginary part of (6) is the rotated phase, which is to be eliminated using an estimated phase. Since the phase rotation induced by the transmitter laser PN will be fully compensated, we are mainly interested in the residual PRT from the LO laser after phase estimation, which can be expressed as

\[
\Phi(k) = \frac{1}{N} \sum_{n=0}^{N-1} \phi(n+D_p) - \phi_n - \phi_k
\]

where \( \phi_{est} \) is the estimated LO laser phase rotation which is the average phase rotation of the PS’s. \( \phi_n \) is the noise distortion, which is usually small and can be approximated as a Gaussian random variable with a zero mean and a variance of \( \phi_n = (\sigma_{\phi_n}^2 + \sigma_{\phi_n}^2)/2N_p \), of which \( \sigma_{\phi_n}^2 \) is the variance of the ICI and \( N_p \) is the number of PS’s. The PRT can also be approximated as a zero-mean Gaussian random process, since \( \phi_r(n) \) is a Gaussian random variable with a zero mean.

For UD-PS’s, we consider the extreme case in which all subcarriers are used as PS’s. In section 4 we will show using simulation that the PRT variance derived from this case is almost the same as in practical systems which only employ part of the subcarriers, e.g. 10%, as PS’s. When all subcarriers are used, the estimated phase is given by

\[
\phi_{est} = \frac{1}{2N} \sum_{i=1}^{N} \left( \frac{1}{N} \sum_{n=0}^{N-1} \phi(n+D_p) \right)
\]

Therefore the variance of the PRT for the uniform distribution case is derived as
\[ \sigma_{\text{PRT},k}^2 = E \left\{ \left[ \phi_k \right]^2 \right\} = E \left\{ \frac{1}{N} \sum_{n=0}^{N-1} \phi_k \left( n + D_k \right) - \frac{1}{N} \sum_{n=0}^{N-1} \phi_k \left( n + D'_k \right) \right\} \]

\[ = \sigma^2 \left\{ \sum_{n=0}^{N-1} \left( \frac{1}{N} A(i) - \frac{1}{N} A(i) \right)^2 + \sum_{n=0}^{N-1} \left( \frac{1}{N} A(i) \right)^2 \right\} + \frac{\sigma_{\text{PN}}^2}{2N_p} \]  

(9)

where

\[ A(i) = \frac{1}{N} \sum_{n=0}^{N-1} \max \left( 0, \min \left( N, \left\{ D_k - i + n + 1 \right\} \right) \right) \]  

(10)

and \( \sigma^2 = 2\pi \beta T_s \) is the variance of the PN noise within one sample duration. \( \left\lceil a \right\rceil \) represents the smallest integer that is larger than \( a \). \( \Delta D = D_k / (k-1) \) is the dispersion-induced walk-off between adjacent subcarriers. The detailed derivation is shown in the Appendix.

For C-PS’s, we consider the case where only the center subcarrier is used for phase estimation. Again, it will be shown that the derived PRT variance is an excellent match to the case when multiple PS’s are centered. The estimated phase can be expressed as

\[ \phi_{\text{est}} = \frac{1}{N} \sum_{n=0}^{N-1} \phi_k \left( n + D_k (N/2) \right) \]  

(11)

And the PRT variance becomes

\[ \sigma_{\text{PRT},k}^2 = E \left\{ \left[ \phi_k \right]^2 \right\} = E \left\{ \frac{1}{N} \sum_{n=0}^{N-1} \phi_k \left( n + D_k \right) - \frac{1}{N} \sum_{n=0}^{N-1} \phi_k \left( n + D_k (N/2) \right) \right\} \]

\[ = \sigma^2 \left\{ \frac{1}{3} M_1^2 + M_0 M_2^2 + M_2 \right\} + \frac{\sigma_{\text{PN}}^2}{2N_p} \]  

(12)

where \( M_0 = \min(N, D_k') \) and \( M_1 = \max(N, D_k') \) with \( D_k' = \left| D_k - D_k (n_{\text{min}}) \right| \). Note that for both cases, \( N_p \) is determined by the actual number of PS’s used. The detailed derivation is also shown in the Appendix.

3.2. Closed-form expression for BER performance

Using the derived \( \sigma_{\text{PRT},k}^2 \), the probability density function (PDF) of the Gaussian distributed phase rotation \( \theta \) for the \( k \)th subcarrier is given as

\[ f_{\text{est}}(\theta) = \frac{1}{\sqrt{2\pi \sigma_{\text{PRT},k}^2}} e^{-\frac{\theta^2}{2\sigma_{\text{PRT},k}^2}} \]  

(13)

Note that the dispersion has no impact on the power degradation \( \alpha_k \) and the ICI power \( \sigma_{\text{ICI},k}^2 \), because both are only determined by the symbol length for each subcarrier when the laser linewidth is fixed. Therefore, they are common to all subcarriers, and with the small PN assumption, they can be approximated as [20, 21]

\[ \alpha_k = E \left\{ \left| H(0) \right|^2 \right\} \approx 1 - \frac{\pi \beta T_s}{3} \]  

(14)

and
ICI_k = E[(UCI(k))^2] \approx \frac{\pi \beta \sigma \Delta}{3} \quad (15)

where \( \beta \) is the overall linewidth of the transmitter and the LO lasers, and equals 2\( \beta \) if identical lasers are used at both sides. Assume that \( \alpha \), \( \sigma_{ICI,k} \) and ASE noise are mutually independent and follow the Gaussian distribution, the signal-to-interference-plus-noise ratio (SINR) for the \( k \)th subcarrier can be derived as

\[
\text{SINR}_k = \frac{\alpha_k}{\sigma_{ICI,k}^2 + \frac{1}{\gamma_k}}
\]  

(16)

where \( \gamma_k \) is the signal-to-noise ratio for the \( k \)th subcarrier, which is also common to all subcarriers. Therefore, the SINR is actually independent on the subcarrier index \( k \).

For QPSK modulation, the BER of the \( k \)th subcarrier can be derived as [15, 21]

\[
\text{BER}(k) = \frac{1}{2} \int_{-\infty}^{\infty} \left[ \text{erfc} \left( \sqrt{\text{SINR}_k \cos^2 \left( \frac{\pi}{4} + \theta \right)} \right) + \text{erfc} \left( \sqrt{\text{SINR}_k \sin^2 \left( \frac{\pi}{4} + \theta \right)} \right) \right] \cdot f_\theta(\theta) d\theta
\]  

(17)

and then by averaging \( \text{BER}(k) \) over all subcarriers, the overall BER can be obtained.

4. Numerical results and discussion

To evaluate the proposed theories and further study the impact of the DEPN on RGI CO-OFDM systems, extensive system simulations have been conducted. The simulated system is a single polarization with 56 Gb/s QPSK modulated input symbols, which corresponds to a 112 Gb/s polarization-multiplexed RGI CO-OFDM system. The oversampling factor \( = N_c/N_s \) is fixed at 1.6 for all FFT sizes used in this work. For the fiber channel, only CD with dispersion parameter \( D = 17 \text{ ps/(nm.km)} \) and ASE noise is taken into consideration. The transmitter and LO lasers have the same linewidth. 10% of the subcarriers (included in \( N_c \)) are used as PS’s for phase estimation. For simplicity, the first-stage OFDE is set to completely compensate CD and PMD, so that no CP is added at the transmitter and OFDM channel estimation can be omitted.

4.1. The effect of PRT

Figure 4 shows the normalized variance of the PRT for both UD-PS’s and C-PS’s with and without ASE noise. Note that for analytical results, all subcarriers are used as PS’s for UD-PS’s in (9) and only the center subcarrier is used for C-PS’s in (12), while for the simulation results, 8 subcarriers are used in both cases. It can be seen that without ASE noise, analytical
results from (9) and (12) match the simulation results, even though different numbers of PS’s are used to calculate the estimated phase $\phi_{\text{est}}$ in (7). When ASE noise is added, a small mismatch in PRT variance appears for only the subcarriers around the center. For both UD-PS’s and C-PS’s, we find that subcarriers on the edges have a much larger PRT variance than those around the center. This is because the PS’s are symmetrically distributed so that the subcarriers around the center are more correlated to the estimated phase obtained from the PS’s. The PRT difference between center subcarriers and edge subcarriers can be observed from the constellations shown in Fig. 5. In addition, the PRT variance of subcarriers on the edge is larger for C-PS’s than for UD-PS’s, while the subcarriers around the center have a larger PRT variance for UD-PS’s than for C-PS’s.

Figure 6 presents the average variance of the PRT and the ICI versus the number of subcarriers $N_c$. First, the analytical results are well matched to the simulation results for both PRT and ICI variance. When $N_c$ decreases, the simulated PRT variance becomes a little larger than the analytical results, which is due to the underestimation of the noise distortion in (9) and (12) when $N_p$ is small (i.e. only a few PS’s are employed). Furthermore, UD-PS’s exhibits a smaller average PRT variance than C-PS’s, implying a better BER performance, which will be shown later. As $N_c$ increases, the ICI becomes proportionally larger, while the PRT is reduced. The effect of the latter includes two factors: 1) with the same dispersion, the correlation of the PN applied on each subcarrier becomes larger as $N_c$ increases, leading to a decreased PRT; 2) The number of PS’s $N_p$ increases with $N_c$, but the SNR for each PS remains the same, so the last term in (9) and (12) is reduced. Therefore, with the specific transmission factors such as the fiber length and laser linewidth, there would be an optimal $N_c$, balancing the effects of PRT and ICI.

4.2. System performance

In Fig. 7(a) the BER performances obtained from both the analytical BER estimator (17) and MC simulation are depicted. It is found that Eq. (17) underestimates the BER, especially when the ICI of the PN becomes predominant for high SNR. This disagreement results from the data pattern effect in the ICI component of (4) [15], which implies that the ICI noise is not
Gaussian distributed and thus produces a larger BER. To improve the accuracy of the BER estimator, the distribution of the ICI should be identified, and the t-distribution has been shown to be a good fit [16]. An accurate BER estimator employing well-matched distribution of the ICI is beyond the scope of this work, since we are mainly focusing on the PRT effect. From Fig. 7(a), it is also found that the system with UD-PS’s performs better than that with C-PS’s as predicted in section 4.1. The required SNR difference at a BER = 10^{-3} is approximately 0.5 dB and it becomes larger as the SNR increases. In Fig. 7(b), it first can be seen that the BER estimator is accurate in terms of the PRT effect, since the BER degradation caused by increasing the CD for analytical results is identical to simulation results. We also find that systems with UD-PS’s and C-PS’s have the same BER in the back-to-back scenario (L = 0 km). And as L increases, the former outperforms the latter gradually. Therefore, for the rest of this paper, only the system with UD-PS’s is considered for our further studies.

![Fig. 7. BER versus (a) SNR with L = 3200 km and (b) transmission distance with SNR = 11 dB, β = 1 MHz for both systems.](image)

![Fig. 8. Required SNR at BER = 10^{-3} versus transmission distance L with (a) different numbers of subcarriers N_c with β = 1 MHz and (b) varying linewidths β and N_c = 80.](image)

As discussed in section 4.1, OFDM signals with large N_c’s are more tolerant to PRT but more vulnerable to ICI. It should be noted that the ICI is caused by both the transmitter and receiver lasers while the PRT is only caused by the receiver laser. It is shown in Fig. 8(a) that under our simulation conditions, OFDM signals with N_c = 80 achieve the best performance with the transmission distance L < 4800 km when β = 1 MHz. The system with N_c = 40 requires a larger SNR at a BER = 10^{-3} than those with N_c = 80 and N_c = 160 even under back-to-back conditions, because with a fixed percentage of PS’s it uses a smaller number of PS’s resulting in a larger noise interference for phase estimation. Furthermore, it exhibits a larger SNR penalty when L is increased. Even though the tolerance to the PRT impairment is improved with an increasing N_c, the system with N_c = 160 performs worse than the system with N_c = 80 because of the larger ICI noise. And the performance degradation due to ICI for signals with N_c ≥ 320 is so severe that the DFB lasers are prohibitive for conventional CO-
OFDM systems. Figure 8(b) shows the required SNR at a BER = $10^{-3}$ as a function of transmission distance $L$ with different amounts of linewidth $\beta$ when $N_c = 80$. At $L = 0$ km, the SNR illustrates the ICI-induced penalty. Due to the small symbol duration of RGI CO-OFDM, the SNR penalty is negligible with $\beta = 100$ kHz, and less than 1 dB even with a linewidth as large as 2 MHz. Within the limit of 4800 km transmission, the SNR penalty due to PRT is acceptable (less than 1.6 dB) for $\beta \leq 1$ MHz. But as $\beta$ is increased to 2 MHz, the PRT becomes catastrophic as the SNR penalty reaches 5 dB at $L = 4800$ km. Therefore, implementation of RGI CO-OFDM systems with low-cost DFB lasers will require PRT compensation.

5. PRT mitigation

The maximum-likelihood phase estimation approach for CO-OFDM was first proposed to remove the noise interference and thus increase the phase estimation accuracy [22]. It has also been applied in single carrier systems to reduce the computation complexity of feedforward carrier recovery algorithms [23]. Here we propose the use of the maximum-likelihood approach to combat the PRT within the RGI CO-OFDM symbols. The novel phase estimation is described as follows:

1. By employing PS’s, we obtain one estimated phase, which is then applied to all the subcarriers.

2. The subcarriers are divided into several groups. Each group then implements an individual maximum-likelihood approach following

$$H_m = \sum_{k=m-N_g+1}^{m} R(k) \hat{R}(k)$$

(18)

$$\phi_m = \tan^{-1}(\text{Im}[H_m]/\text{Re}[H_m])$$

(19)

where $m$ is the index of the group, $N_g$ is the number of subcarriers in each group, $R(k)$ is the received subcarrier after the first stage phase estimation, $\hat{R}(k)$ is the subcarrier after decision, and $\phi_m$ is the estimated phase for each group. We refer to this method as the grouped maximum-likelihood (GML) approach, because the phase estimation is performed for each group. The computation complexity of GML is almost the same as the conventional maximum-likelihood phase estimation, which is reasonably low for implementation considerations [23], as long as the total number of groups is small.

Because of the nature of laser PN and fiber CD, the variance of the phase rotation difference between two subcarriers is determined by their frequency spacing. Therefore, if we divide the subcarriers in order of subcarrier index into several groups (e.g. subcarriers 1 to 10 are in one group, and subcarriers 11 to 20 are in another group, etc.) and compensate the phase using information from each group, the PRT will be significantly reduced. Ideally, the phase rotation of each individual subcarrier can be extracted if each group is made to have only one subcarrier, and then PRT will be completely eliminated. However, in order to remove both the noise interference and the possible wrong phase estimation from an incorrect decision, averaging over the phase information from multiple subcarriers is required.

As shown in Fig. 9(a), the optimal number of subcarriers in each group $N_p$ is 20 (4 groups) for both scenarios, using a laser linewidth 1 MHz and 2 MHz respectively. The required SNR increases when smaller $N_p$ is used because the phase information acquired from the subcarriers in one group is not enough to remove the noise interference and the incorrect decision, which will lead to unreliable phase estimation. On the other hand, when $N_p$ is too large, the PRT power included in each group will also be too large so that the penalty induced by PRT is mitigated less. Fig. 9(b) shows the system performance as a function of the number of PS’s $N_p$, when 80 subcarriers are divided into four groups with $N_p = 20$. The performance is improved as $N_p$ increases, because more accurate decisions can be made due to the
improved reliability of the phase estimation in the first stage. It should be noted that UD-PS’s are applied rather than C-PS’s, since the former achieves better performance as shown earlier. Eventually, considering the tradeoff between the overhead and performance improvement, we choose 4 PS’s because the improvement is limited to less than 0.1 dB and 0.2 dB using more PS’s for a linewidth of 1 MHz and 2 MHz respectively.

Fig. 9. Required SNR at BER $= 10^{-3}$ versus (a) the number of subcarriers in each group $N_g$ with $N_p = 4$ and (b) number of PS’s $N_p$ with $N_g = 20$.

Figure 10 compares the BER performance of the system with and without GML phase estimation. Note that 80 subcarriers are used for all systems and the system without GML phase estimation employs 8 subcarriers (10%) as PS’s. The performance of the system with one group GML at $L = 3200$ km is also shown, of which the BER is lowered compared to the system without GML. This is because the noise interference is almost completely removed as in the conventional CO-OFDM [22], which means the performance degradation is only caused by the PRT impairment. However, this improvement is limited when compared to the system with the 4-group GML, which implies the dominance of the PRT impairment with respect to the noise interference. We observe a 0.7 dB and 1.7 dB SNR improvement for the system with 4-group GML at BER $= 10^{-3}$, compared to the system without GML, for a linewidth of 1 MHz and 2 MHz respectively. It should be noted that the PS overhead is also reduced from 10% to 5%. Moreover, by comparing to the back-to-back scenario, we observe that the residual PRT penalty, which is 0.25 dB and 0.6 dB for a linewidth of 1 MHz and 2 MHz respectively, becomes acceptable for practical implementation. However, a more advanced PRT compensation scheme is required to further reduce the PRT penalty and as well as PS’s overhead, in order to make RGI CO-OFDM as competitive as single carrier systems in terms of laser PN tolerance.

Fig. 10. Illustration of the effectiveness of GML phase estimation with (a) $\beta = 1$ MHz and (b) $\beta = 2$ MHz.
6. Conclusions

In this work, we show the effect of the dispersion-enhanced phase noise (DEPN) in reduced-guard-interval CO-OFDM systems. The variance of the subcarrier-dependent phase rotation term (PRT) is analytically presented when employing two different distribution patterns of pilot subcarriers (PS). And one approximate bit error ratio (BER) estimator is presented as well to quantify the performance degradation caused by DEPN. Numerical simulations are performed to verify the analytical results, and further discussions are made based on these results. Finally, we propose a grouped maximum-likelihood (GML) phase estimation, which enables the mitigation of the DEPN impairment with a 0.7-1.7 dB SNR improvement at BER $= 10^{-3}$. This improvement enables the use of DFB lasers in RGI CO-OFDM systems.

APPENDIX

The phase shift for each sample in discrete time domain for the $k$th subcarrier with respect to the first sample in the 1st subcarrier ($n = 0, k = 1$) can be expressed as [15, 20]

$$
\phi_i(n) = \sum_{i=0}^{N_c-1} v(i)
$$

where $v(i)$ are mutually independent, and each follows Gaussian process with a zero mean and a variance of $\sigma^2 = 2\pi\beta T_c$.

1). For UD-PS’s, Eq. (9) can be derived as follows:

$$
\sigma_{\text{dev.}}^2 = E \{ |\Phi(k)|^2 \} = E \left\{ \frac{1}{N_c} \sum_{i=0}^{N_c-1} \left[ \frac{1}{N_c} \sum_{i=0}^{N_c-1} \left( \frac{1}{N_c} \sum_{i=0}^{N_c-1} \phi_i(n+D_k) - \phi_i \right) \right] \right\}
$$

$$
= E \left\{ \frac{1}{N_c} \sum_{i=0}^{N_c-1} \left( \frac{1}{N_c} \sum_{i=0}^{N_c-1} v(i) - \phi_i \right) \right\}
$$

$$
= E \left\{ \frac{1}{N_c} \sum_{i=0}^{N_c-1} v(i) + \frac{1}{N_c} \sum_{i=0}^{N_c-1} [N - (i - D_k)] v(i) - \frac{1}{N_c} \sum_{i=0}^{N_c-1} A(i) v(i) - \phi_i \right\}
$$

where $A(i)$ is already given by Eq. (10). Since $v(i)$ and $\phi_i$ are also mutually independent, and the variance of $\phi_i$ is given by $\phi_i = (\sigma_{\text{dev}}^2 + \sigma_{\text{ase}}^2)/2N_c$, we finally arrive at

$$
\sigma_{\text{dev.}}^2 = \sigma^2 \left\{ \frac{1}{N_c} \sum_{i=0}^{N_c-1} [1 - \frac{1}{N_c} A(i)]^2 + \frac{1}{N_c} \sum_{i=0}^{N_c-1} \left[ \frac{1}{N_c} A(i) - \frac{1}{N_c} \sum_{i=0}^{N_c-1} A(i) \right]^2 + \frac{1}{N_c} \sum_{i=0}^{N_c-1} \left( \frac{1}{N_c} A(i) \right)^2 \right\} + \frac{(\sigma_{\text{dev}}^2 + \sigma_{\text{ase}}^2)}{2N_c}
$$

2). For C-PS’s, Eq. (11) is derived as below:
\[\sigma_{\text{PSF},k}^2 = E\left\{\Phi(k)\right\}^2 = E\left\{\frac{1}{N} \sum_{n=0}^{N-1} \phi_n(n + D_k) - \frac{1}{N} \sum_{n=0}^{N-1} \phi_n(n + D_{k-1/2}) - \phi_k\right\}^2\]

\[= E\left\{\frac{1}{N} \sum_{n=0}^{N-1} \frac{1}{\sigma^2} \sum_{i=0}^{N-1} v(i) - \frac{1}{N} \sum_{n=0}^{N-1} \frac{1}{\sigma^2} \sum_{i=0}^{N-1} v(i) - \phi_k\right\}^2\]

\[= E\left\{\frac{1}{N} \sum_{n=0}^{N-1} \sum_{i=0}^{N-1} v(i) - \phi_k\right\}^2\]

\[= E\left\{\frac{1}{N} \sum_{n=0}^{N-1} \left[ q[v(q) + v(N + D'_k - q)] + \frac{M_k}{N} \sum_{p=M_k}^{M_k+1} v(p) - \phi_k\right]\right\}^2\]

\[= \frac{\sigma^2}{N^2} \left[ \frac{(M_a - 1)M_a (2M_a - 1)}{6} + M_a^2 (M_b - M_a + 1) \right] + \frac{(\sigma_{\text{PSF},k}^2 + \sigma_{\text{PSF},k}^2)}{2N_p}\]

\[= \frac{\sigma^2}{N^2} \left[ -\frac{1}{3} M_a^2 + M_a^2 + \frac{M_a}{3} \right] + \frac{(\sigma_{\text{PSF},k}^2 + \sigma_{\text{PSF},k}^2)}{2N_p}\]

where \(D'_k = \left| D_k - D_{k-1/2} \right|\), \(M_a = \min(N, D'_k)\) and \(M_b = \max(N, D'_k)\).