Wide-Range, Low-Complexity Frequency Offset Tracking Technique for Single Carrier Transmission Systems

Meng Qiu, Qunbi Zhuge, Xian Xu, Mathieu Chagnon, Mohamed Morsy-Osman, and David V. Plant
Dept. of Electrical and Computer Engineering, McGill University, Montreal, QC, H3A 2A7, Canada
E-mail: meng.qiu@mail.mcgill.ca

Abstract: We propose a wide-range, low-complexity frequency offset tracking algorithm for single carrier transmission systems. Its effectiveness is validated numerically and experimentally in a 112 Gb/s DP-QPSK system.

OCIS codes: (060.1660) Coherent communications; (060.5060) Phase modulation

1. Introduction
In coherent transmission systems with a free-running local oscillator (LO), frequency offset (FO) between the transmit laser and the LO is an inevitable problem. It introduces phase rotation on the received symbols, which translates to phase errors in the carrier phase recovery stage. Several approaches have been proposed to estimate and compensate for FO in signal carrier transmission systems. In [1-3], the authors describe several spectrum based methods. However, these algorithms require a Fourier transform of the received signals, so generally their computational complexity is high. Other approaches utilize the phase increment between adjacent symbols [4-6]. But phase operations are still required for each symbol to make the algorithm tolerant to large FO. Given that the computational resources are limited at the receiver, further reduction of the computational complexity is desirable.

In addition, the laser frequency continues to drift, so the FO is not constant in practical systems [7]. Therefore, merely getting a global FO for a long received sequence is not enough, and accurately tracking the FO change is necessary.

In this paper, we propose a low-complexity FO tracking algorithm which is capable of tracking the FO change with high accuracy. In simulation, our algorithms gives <1 MHz estimation error when the FO drift rate is 2 MHz/μs. In addition, the algorithm is able to correct FO drifts up to 200 MHz/μs with little degradation. Then we demonstrate experimentally in a 112 Gb/s dual-polarization (DP) quadrature phase-shift keying (QPSK) system that the algorithm successfully detects a FO drift of more than 80 MHz. Via accurate tracking and dynamic compensation of FO, the algorithm maintains good system performance in the presence of FO drift, while in the configuration without a tracking mechanism, system performance degrades when FO drift becomes significant.

2. Principle
The proposed algorithm includes two parts: a) the initial FO estimation; and b) FO drift tracking. The initial FO estimation gives the overall frequency offset of the received sequence. To achieve this we use the training-symbol-aided method described in [4]. During tracking, we divide the received sequence into multiple blocks, with the block length determining the time resolution of the tracking. Before the (n+1)th block is processed for phase recovery, it is pre-compensated by the estimated FO of the previous block f_{0,n}. Then the residual FO, Δf_{0,n}, is estimated for the current block, as shown in Fig.1, and is added to f_{0,n} to get the next FO estimation f_{0,n+1}. Finally the new FO estimation is used for the pre-compensation of the next block, and the operations above are repeated. This process gives good approximation when we choose the block length properly to make the FO drift inside a block sufficiently small.

To estimate Δf_{0,n}, we calculate the phase increment of every P symbols and use:

\[ \varphi(k + P) - \varphi(k) = 2\pi f_{0,n} PT_{sym} + \varphi_{noise} \]  

Here \( \varphi \) is the phase after the removal of modulation, \( T_{sym} \) is the symbol duration, \( \varphi_{noise} \) includes the random phase errors related to laser phase noise and additive white Gaussian noise (AWGN), both having zero mean [8]. This term can be neglected after we average the phase increment (the left side of Eq.1) within a block, so the phase increment is mainly attributed to the FO-induced phase rotation. To avoid ambiguity, the phase increment should be confined within \([−\pi, \pi]\), so the spacing between the two investigated symbols is limited in the conventional phase increment algorithms. As an improvement, we perform the FO pre-compensation described above. By doing this, Δf_{0,n} is only several MHz if the FO changes smoothly and the FO variation inside a block is sufficiently small. So even if we calculate the phase increment every 50 symbols, the phase change is still much smaller than \( \pi \) in a high
baud rate system. Compared with the methods in [4-6] which calculate between adjacent symbols, our approach significantly reduces the number of computations in the FO estimation stage.

3. Setup, results and discussions

In simulation, a 28-GBd QPSK sequence with $1.12 \times 10^6$ symbols is generated and contaminated by the laser phase noise and AWGN. The laser phase noise used results from a total system linewidth of 200 kHz, corresponding to the ~100 kHz linewidths of the ECLs used at both the transmitter and the LO. Then we set a FO variation manually and impose the FO-induced phase rotation to the symbols. Transmission impairments and other effects are not considered, and the symbols are demodulated through a phase-locked loop (PLL) and differential decoding. Fig.2 shows a specific case when the overall FO is 1GHz, the FO drift rate is 2 MHz/$\mu$s and the block length is $10^4$. This variation of FO is successfully tracked by our algorithm with estimation errors smaller than 1 MHz. Fig.3 demonstrates the algorithm’s tolerance to a high FO drift rate. The bit error rates (BER) in this figure are obtained in the optimized configuration for each case. With the aid of FO tracking, FO drifts up to 200 MHz/$\mu$s can be compensated with minimal degradation. In contrast, the PLL without FO tracking collapses when the FO changes at a high rate.

Next we conduct experiments in a 112 Gb/s DP-QPSK system. Fig.4 illustrates the transmission setup. ECLs serve as the lasers at both the transmitter and the LO. The signals provided by the digital-to-analog converters (DACs) drive an IQ-modulator to perform electrical-to-optical conversion. The DP signal is obtained by splitting the signal into two polarizations, delaying one polarization and re-combining them. Next the signal is boosted to the optimal launch power before it is sent to the re-circulating loop. The loop consists of 320 km of standard single mode fiber, and an EDFA with noise figure of 5 dB is employed every 80 km to compensate for fiber losses. The output from the loop is filtered, pre-amplified and filtered again. After coherent detection, the electrical signals are digitized by two 80GS/s real-time scopes. Finally the digital signal is processed offline in MATLAB. The processing is performed on 8 parallel streams, where channel impairment compensation, FO estimation/compensation and carrier phase recovery are performed, and differential coding/decoding is employed to avoid cycle slip. We sampled a sequence containing $7.02 \times 10^6$ symbols, which corresponds to 250.71 $\mu$s in the 28 GBd system. The block length is 2000 in each parallel stream, which enables a time resolution of 0.571 $\mu$s considering the parallelism is 8. The value of $P$ in Eq.1 is 50. This means we calculate the phase increment once every 50 symbols, reducing the number of computational steps to 1/50 compared to the previous phase increment estimation approaches.

The estimated FO based on the proposed algorithm is depicted in Fig.5. During a time duration of 250 $\mu$s, the
FO drifts more than 80 MHz. The red diamond markers represent the FO of the discrete data sections, which are estimated by the spectrum-based method in [1]. From the comparison, we can see the results based on our approach match well with the values given by the spectrum-based approach.

Next system performance with and without FO tracking is compared. The same received symbol sequence is loaded to the two configurations and processed. For the system without FO tracking, the sequence is only compensated by the initial FO values estimated from training symbols, and there is no mechanism to detect the FO change afterwards. Fig.6 summarizes the BER versus sequence length relationship. The BERs are obtained in the optimal configuration for each case. Specifically, if the tracking mechanism is not included, the system performance gradually deteriorates when the processed data sequence becomes longer. This is because the entire sequence is merely compensated by the first estimated FO. After frequency drifts, there will be residual FO in the rest of the sequence, which leads to a rotation of the QPSK constellation in the complex plane and causes errors. In contrast, with FO tracking, both the overall FO and the FO variation are corrected, so a low BER is maintained even when the processed sequence becomes long and FO drift inside the sequence becomes significant (>80 MHz).

In terms of complexity, we compare the number of computations required for the FO estimation, which is summarized in Table.1. The computation required for removing the modulation and FO compensation is not taken into account. In the spectrum-based methods, a Fourier transform is required. With a fast Fourier transform (FFT), the number of computations is on the order of $O(N \log N)$ for each symbol, where $N$ is the number of points involved in FFT. For the methods utilizing the FO-induced phase rotation between adjacent symbols, every symbol is involved in calculation and $\approx 2$ real operations are required for each symbol. In our algorithm, the calculation is performed every $P$ symbols, so for each symbol, the required number of computations is reduced to $\approx 2/P$, which amounts to a substantial computational saving.

<table>
<thead>
<tr>
<th>Algoritms</th>
<th>Spectrum-based algorithm</th>
<th>Conventional phase increment estimation algorithm</th>
<th>The proposed algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computational cost</td>
<td>$\sim \log_2 N$ complex multiplications and additions</td>
<td>$\approx 2$ real operations</td>
<td>$\approx 2/P$ real operations</td>
</tr>
</tbody>
</table>

4. Conclusion

We propose a wide-range, low-complexity FO tracking algorithm. It significantly reduces the computational complexity compared with spectrum-based approaches and conventional phase increment estimation approaches. In simulation, it gives accurate FO estimations with small errors, and it is able to compensate FO drifts up to 200 MHz/µs. Experimentally we successfully tracked the FO of a data sequence having >80 MHz of FO drift. Finally, we show that the algorithm maintains good system performance in the presence of FO drift, which is in contrast with the significant degradation in the configuration without tracking mechanisms.

5. Reference