Analytical solution for the design of planar EBG structures with spurious-free frequency response

I. Arnedo [1,2], J. D. Schwartz [2], I. Arregui [1], M. A. G. Laso [1], D. V. Plant [3], J. Azaña [2], and T. Lopetegi [1]

[1] *Electrical Engineering Department, Public University of Navarre, E-31006 Pamplona, Naverre, Spain.*
israel.arnedo@unavarra.es

[2] *Institut National de la Recherche Scientifique – Energie, Matériaux et Télécommunications (INRS-EMT), Montréal, QC, H5A 1K6, Canada.*

[3] *Photonics Systems Group, Department of Electrical and Computer Engineering, McGill University, Montreal, QC, H3A 2A7, Canada.*

***Abstract*— In this paper, an analytical solution is obtained for the design of Electromagnetic Bandgap (EBG) structures that feature a single stopband without the spurious rejected bands present in the conventional EBG devices at the harmonics of the design frequency. The design technique is applied to a two-port microstrip EBG circuit and to a four-port coupled-line EBG device in stripline technology obtaining very good results. The implementation of the EBG structure in coupled-lines allows for the redirection of the reflected signal to the coupled-port producing a device with input port matched at all frequencies.**

I. INTRODUCTION

In the last years, the field of periodic structures for electromagnetic waves has been a subject of great interest for the microwave community due to the introduction of the novel and fruitful concepts of Electromagnetic Bandgap (EBG) structure, metamaterial, and negative-parameter structure. These novel concepts have given rise to new design approaches that allow us to obtain improved microwave devices and even new components for novel applications. There has been an important research effort to make these novel periodic structures compatible with planar microwave technologies, making easier their practical implementation and widening significantly their scope of potential applications. Specifically, periodic structures of the EBG-type have been reported as very useful to improve the behaviour of planar circuits and antennas by introducing stopbands that forbid the propagation of electromagnetic waves in the unwanted frequency bands and directions. Actually, they have found very promising applications including the implementation of filters and resonators, the improvement of the efficiency and radiation pattern of antennas, and harmonic tuning in power amplifiers, oscillators and mixers [1]. Although the frequency response of a conventional EBG structure features a wide and deep stopband as intended, it also includes spurious rejected bands at the harmonics of the design frequency that can be detrimental in many applications. Moreover, in most of the cases, the design of these structures is laborious because it requires full-wave electromagnetic simulations and a trial-and-error method. In this paper, an analytical solution for the spurious-free EBG structure with full control over all the design parameters is proposed. The solution is based on the coupled-mode theory [2], [3] and it is applied to design conventional single-line structures as well as coupled-lines where the rejected frequency band is redirected to the coupled port, which can be of high interest in many applications.

II. ANALYTICAL DESIGN OF THE EBG STRUCTURE

As it is explained in [2], [3], using the coupled-mode theory and assuming single-mode operation, the electromagnetic behavior of an EBG structure implemented in planar microwave technology can be characterized through the coupling coefficient, $K(z)$, between the forward and backward travelling-waves associated to the operation mode. If the EBG structure is periodic along the propagation axis z with period Λ, then the $K(z)$ produced by the perturbation is also periodic with the same period and, hence, it can be expanded in a Fourier series as:

$$K(z) = \sum_{n=-\infty}^{n=\infty} K_n \cdot e^{j\cdot\frac{2\cdot\pi}{\Lambda}\cdot n\cdot z} \tag{1}$$

$$K_n = \frac{1}{\Lambda}\cdot\int_{\Lambda} K(z)\cdot e^{-j\cdot\frac{2\cdot\pi}{\Lambda}\cdot n\cdot z}\cdot dz \tag{2}$$

Assuming that the operation mode is TEM or quasi-TEM (as it happens in most of the planar circuit technologies), analytical expressions can be obtained for the central frequency, f_c, rejection level, $|S_{21}|_{\min}$, (or equivalently reflexion level, $|S_{11}|_{\max}$), and bandwidth between zeros, BW, of the stopbands produced by the EBG structure (see [2]):

$$f_c = \frac{c\cdot n}{2\cdot\Lambda\cdot\sqrt{\varepsilon_{eff}}} \tag{3}$$

$$|S_{21}|_{\min} = \operatorname{sech}(|K_n|\cdot L);\ |S_{11}|_{\max} = \tanh(|K_n|\cdot L) \tag{4}$$

$$BW = \frac{c \cdot |K_n|}{\pi \cdot \sqrt{\varepsilon_{eff}}} \cdot \sqrt{1 + \left(\frac{\pi}{|K_n| \cdot L}\right)^2} \quad (5)$$

where c is the speed of light in vacuum, L is the device length, and ε_{eff} is the mean value of $\varepsilon_{eff}(z)$ in a period, calculated as $\varepsilon_{eff} = \left(\frac{1}{\Lambda} \cdot \int_{\Lambda} \sqrt{\varepsilon_{eff}(z)} \cdot dz\right)^2$.

As it can be seen, the n-*th* coefficient of the Fourier series, K_n, is univocally associated to the n-*th* stopband and, therefore, to obtain a spurious-free EBG structure that features only the fundamental stopband, all the coefficients must be equal to zero except for $K_{\pm 1}$. This results in a sinusoidal coupling coefficient with the generic expression:

$$K(z) = A \cdot \cos\left(\frac{2 \cdot \pi}{\Lambda} \cdot z + \theta\right) \quad (6)$$

where $K_{\pm 1} = A/2 \cdot e^{\pm j \cdot \theta}$ and $K_n = 0$ for $|n| \neq 1$, which produces an EBG structure with a single stopband with the following design parameters (obtained from (3) to (5)):

$$f_c = \frac{c}{2 \cdot \Lambda \cdot \sqrt{\varepsilon_{eff}}} \quad (7)$$

$$|S_{21}|_{\min} = \operatorname{sech}(A \cdot L/2); \; |S_{11}|_{\max} = \tanh(A \cdot L/2) \quad (8)$$

$$BW = \frac{c}{\pi \cdot \sqrt{\varepsilon_{eff}}} \cdot \sqrt{\left(\frac{A}{2}\right)^2 + \left(\frac{\pi}{L}\right)^2} \quad (9)$$

The expressions that relate the physical dimensions of the device and the desired coupling coefficient will depend on the technology used [3]. For our case of TEM or quasi-TEM operation mode, the characteristic impedance $Z_0(z)$ can be used [2], [3]:

$$K(z) = -\frac{1}{2} \cdot \frac{1}{Z_0} \cdot \frac{dZ_0}{dz} \quad (10)$$

and, therefore, the characteristic impedance profile needed to obtain the designed EBG structure with the coupling coefficient given by (6) is:

$$Z_0(z) = Z_0(0) \cdot e^{-\frac{A \cdot \Lambda}{\pi} \cdot \left[\sin\left(\frac{2 \cdot \pi}{\Lambda} \cdot z + \theta\right) - \sin(\theta)\right]} \quad (11)$$

where $Z_0(0)$ is the input port characteristic impedance.

III. EBG Structure in Microstrip

In order to demonstrate the accuracy of the analytical design method proposed in the previous section, a spurious-free EBG structure with stopband central frequency, f_c = 3.6 GHz, rejection level $|S_{21}|_{\min}$ = -32 dB, and rejection bandwidth BW = 1.5 GHz, is designed in microstrip technology. The substrate employed is a Rogers RO3010, with ε_r = 10.2 ± 0.3 and h = 1.27 mm. By using (7)-(9), the necessary parameters to fix the EBG characteristic impedance profile given by (11) can be obtained: Λ = 15.7 mm, A = 70 m^{-1}, and $L = 8 \cdot \Lambda$. For convenience, the initial phase is fixed at $\theta = 0$ rad and the input port characteristic impedance at $Z_0(0)$ = 50 Ω. From the analytical characteristic impedance profile, $Z_0(z)$, the necessary strip-width can be obtained using well-known expressions available in classical textbooks, thus concluding the design process, see Fig. 1.

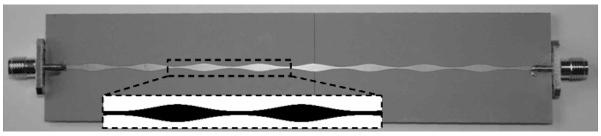
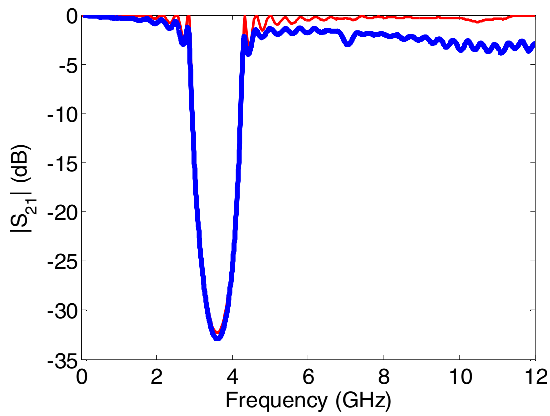
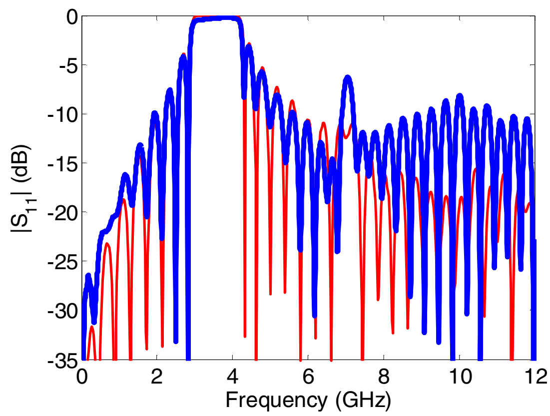


Fig. 1. Conductor strip-width of the designed EBG microstrip structure.



(a)



(b)

Fig. 2. a) $|S_{21}|$ and b) $|S_{11}|$ for the designed EBG microstrip structure: simulation (thin red line) and measurement (thick blue line).

The device was simulated using Agilent ADS Momentum and measured with an Agilent 8722 Vector Network Analyser. The results are depicted in Fig. 2. Very good agreement can be observed between simulations and measurements confirming the presence of a single stopband with complete absence of spurious harmonic rejected bands. The stopband central frequency, rejection level, and rejection bandwidth are f_c = 3.6 GHz, $|S_{21}|_{\min}$ = -32.25 dB, and BW = 1.48 GHz in simulation. On the other hand, the measured values are f_c = 3.6 GHz, $|S_{21}|_{\min}$ = -32.80 dB, and BW = 1.50 GHz. As it can be seen, the agreement between the target, simulated and measured values is very good. The small discrepancies found can be attributed mainly to the fabrication tolerances.

IV. Four-Port EBG Structure in Stripline

A four-port spurious-free EBG structure will be also designed using coupled-lines. Symmetrical coupled-lines are customarily studied employing the even and odd modes of the coupled-line structure. Taking advantage of this mode decomposition, the S-parameters can be calculated as [4]:

$$S_{11}(f) = \frac{\Gamma_e(f) + \Gamma_o(f)}{2} \tag{12}$$

$$S_{31}(f) = \frac{\Gamma_e(f) - \Gamma_o(f)}{2} \tag{13}$$

where $\Gamma_e(f)$ and $\Gamma_o(f)$ are the reflection coefficients of the even and odd mode, respectively, and the port numbers follow the convention of Fig. 3. As it can be seen in (12) and (13), if the design condition $\Gamma_e(f) = -\Gamma_o(f)$ is satisfied, then $S_{11}(f) = 0$ and $S_{31}(f) = \Gamma_e(f)$, producing a device with the input port (#1) matched at all frequencies and with the reflected signal redirected to the coupled port (#3). If the even and odd modes feature the same phase constant, β, for a given frequency, as it happens in our case (stripline technology), then the design condition can be rewritten as $\Gamma_e(\beta) = -\Gamma_o(\beta)$. Applying the sign inversion property reported in [3] for the relationship between $K(z)$ and $\Gamma(\beta)$, the design condition can be finally rewritten as:

$$K_e(z) = -K_o(z) \tag{14}$$

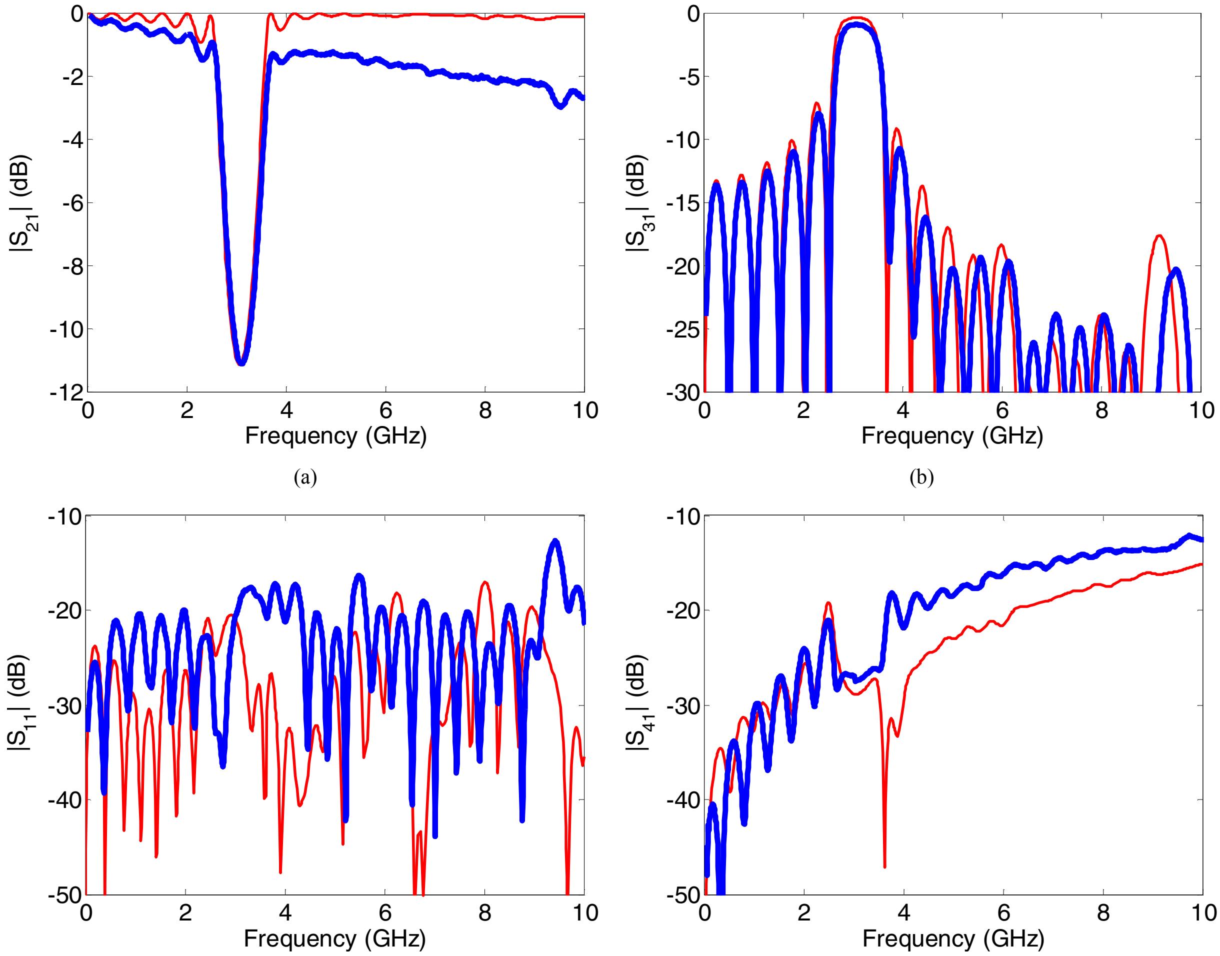
where $K_e(z)$ and $K_o(z)$ are the coupling coefficient for the even and odd modes, respectively.

Therefore, to obtain a spurious-free EBG structure that has its input port matched at all frequencies and the stopband redirected to the coupled port, (6) must be used as $K_e(z)$ and, consequently, the even mode characteristic impedance, $Z_{0,e}(z)$, must follow the profile in (11). Regarding the odd mode, it can be seen using (14) that $K_o(z)$ must be equal to (6) but with opposite sign in the A parameter and, consequently, $Z_{0,o}(z)$ must follow the profile in (11) but also with opposite sign in the A parameter. The four-port EBG structure obtained will have the same design parameters as in the previous section (equations (7)-(9)), but taking into account that since the reflected signal is now redirected to the coupled port $S_{31}(f)$ plays now the role of $S_{11}(f)$ and the input port is matched $S_{11}(f) = 0$, and that $\varepsilon_{eff} = \varepsilon_r$ in stripline technology.

Fig. 3. Conductor strip-width of the designed four-port EBG stripline structure.

In order to demonstrate the design method proposed, a four-port spurious-free EBG structure with stopband central frequency, f_c = 3.1 GHz, rejection level $|S_{21}|_{\min}$ = -11 dB, and rejection bandwidth BW = 1.2 GHz, is designed in stripline technology (using a Taconic CER10 substrate with ε_r = 9.8 ± 0.5 and b = 2.54mm). The resulting parameters to fix the even and odd mode characteristic impedance profiles, $Z_{0,e}(z)$ and $Z_{0,o}(z)$, are Λ = 15.7 mm, A = 42 m^{-1}, $L = 6 \cdot \Lambda$, taking $\theta = \pi/2$ rad and $Z_{0,e}(0) = Z_{0,o}(0)$ = 50Ω for convenience. Using the expressions available in [4], the coupled-line physical dimensions can be obtained from $Z_{0,e}(z)$ and $Z_{0,o}(z)$, concluding the design process, see Fig. 3.

The circuit simulations and measurements are depicted in Fig. 4. A single stopband with complete absence of spurious harmonic rejected bands is obtained. The stopband central frequency is f_c = 3.06 GHz in simulation and 3.11 GHz in measurement, while the rejection level and bandwidth are $|S_{21}|_{\min}$ = -11.14 dB and BW = 1.21 GHz, both in simulation and measurement. As it can be seen, an excellent agreement is achieved again between the target, simulated and measured values, while the input port is matched for the full bandwidth as intended.



(a) (b) (c) (d)

Fig. 4. a) $|S_{21}|$, b) $|S_{31}|$, c) $|S_{11}|$ and d) $|S_{41}|$ for the designed four-port EBG stripline structure: simulation (thin red line) and measurement (thick blue line).

V. Conclusions

An analytical solution for the design of planar EBG structures with spurious-free frequency response has been proposed. Several expressions for the main design parameters have been also provided. The design technique has been successfully tested both in a two-port microstrip device and in a four-port coupled-line circuit in stripline technology. The implementation of EBG structures in coupled-lines allows us to extend significantly their scope of application since the reflected signal is redirected to the coupled port producing a EBG structure matched at all frequencies.

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