Extended-Serial Decoding for Turbo-Coded Data Gathering Sensor Networks

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***Abstract*— We consider a specific type of data gathering sensor networks that can be modeled by a binary chief executive officer problem. We apply turbo codes to encode sensors observations and transmit them to a fusion center over independent binary symmetric channels. It is shown in the literature that the fusion center can exploit the correlation between sensors observations to design a soft-input soft-output (SISO) *global decoder*. Then the fusion center iterates extrinsic information between the *global decoder* and the SISO decoder of the applied error correcting code to jointly estimate the source. Since we consider turbo codes, the joint decoding problem is generalized to the problem of exchanging extrinsic information between three SISO modules. In this paper, we first apply the sum-product algorithm to derive the rules that update extrinsic information for the *global decoder*. Then, we apply extended-serial decoding that is the best known structure for decoders consisting of three concatenated SISO modules. We compare the bit error rate achieved by extended-serial decoding with the one achieved by a separate decoding strategy, where the fusion center separately decodes each sensor's observation and then decides based on the majority of the outputs. Our simulations show that extended-serial decoding performs significantly better than separate decoding.**

Index Terms— Wireless sensor networks, CEO problem, extended-serial decoding, turbo codes.

I. INTRODUCTION

Data-gathering wireless sensor networks (WSNs) have applications in different areas, including environmental and structural monitoring, health care, rescue operations, and disaster recovery [1], [2], [3], [4]. In this work, we consider a data gathering WSN depicted in Fig. 1. Sensors observe independent noisy versions of a source, encode their observation, and transmit to a fusion center (FC) through independent noisy channels. This model is considered in different works including [5], [6], [7]. A well-known Chief Executive Officer (CEO) problem [8], [9], [10] can be applied to model the considered WSN. We are interested in different decoding strategies that the FC could take to estimate the source.

A simple separate decoding strategy suggests that the FC separately decodes each observation and then decides based on the majority of the outputs. In contrast, a joint decoding method is proposed in [11] that regards the correlation between sensors observations as a *global code*, and designs a soft-input soft-output (SISO) *global decoder*[1]. Then, they iterate the extrinsic information between this global decoder and the BCJR decoder [12] of the error correcting code (applied at sensors to encode and transmit observations), to jointly estimate the source. This approach is similar to the problem of turbo equalization [13], [14] where the knowledge of inter-symbol interference is employed to update log-likelihood ratios (LLRs) for the next round of decoding. When sensors apply error correcting codes with a single BCJR decoder[2], e.g. convolutional codes, the iterative decoding approach is well-established [15], [11]. In this case, the BCJR decoder and the global decoder run in serial and exchange their extrinsic information. However, if sensors apply concatenated codes, e.g. turbo codes, that require two concatenated BCJR decoders, the problem is generalized to the problem of exchanging extrinsic information between three SISO modules.

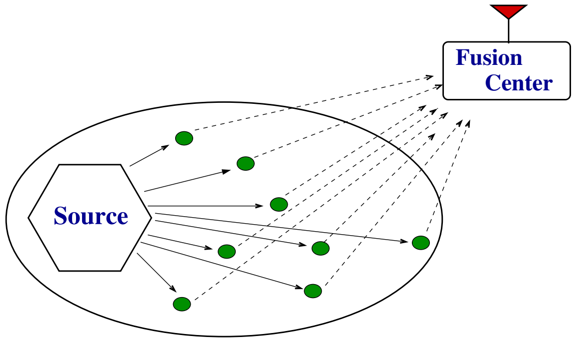

Fig. 1. Schematic of the considered data gathering sensor network.

The problem of exchanging extrinsic information between three SISO modules is considered in the literature and different decoding structures are proposed. An extensive study on different decoding structures [16] shows that an *extended-serial* decoder has superior performance, in which the three SISO decoders run in serial, and each decoder passes its extrinsic information to the other two decoders (Fig. 4). This extended-serial decoder is applied in [17] for extrinsic information transfer chart (EXIT chart) analysis of multiple turbo codes[3]. Note that instead of exchanging extrinsic information between three decoders, it is possible to design a maximum likelihood decoder, as shown in [18]. This will

[1]In [11] sensors observe different correlated sources, whereas in our model they observe noisy versions of the same source. However, these observations are still correlated and the algorithm proposed in [11] can be applied.

[2]Any other SISO decoding scheme can replace the BCJR decoder.

[3]Multiple turbo codes are the class of turbo codes with three or more constituent convolutional codes.

decrease the bit error rate at the expense of increasing the decoding complexity. A pioneer work on joint decoding in sensor networks [5] takes a similar approach, where the whole system is modeled by a graph, and a belief-propagation algorithm runs on this graph.

In this paper, we first derive the rules that update extrinsic information for the global decoder. For this, we assign *variable nodes* [19] to the source and to each sensor observation. Then, we consider the correlation between the source and sensors observations as a *function node* [19] that connects all the variable nodes on a graph. We apply the sum-product update rules [19] on this graph, to find the extrinsic LLRs. After designing the global decoder, we simulate the extended-serial decoding and compare the achieved bit error rate with the one achieved by separate decoding. Our simulations show that extended-serial decoding performs significantly better than separate decoding. The rest of this paper is organized as follows: Section II presents the system model for the considered data-gathering sensor network. In Section III we consider the global decoder and apply the sum-product update rules to derive the equations for the extrinsic information. We also review the concept of extended-serial decoding and show how the FC applies this structure to jointly estimate the source. Section IV is devoted to simulation results and discussions. Section V concludes the paper.

II. System Model

We consider a binary CEO model as shown in Fig. 2. The message sequence $\mathbf{x} = \{x(k)\}_{k=1}^{K}$ is an outcome of an unbiased i.i.d. binary source. N sensors make noisy observations of this message as $\mathbf{y}_n = \{y_n(k)\}_{k=1}^{K} = \mathbf{x} \oplus \mathbf{v}_n$, where $\mathbf{v}_n$ is a binary i.i.d. sequence with $\Pr(v_n(k) = 1) = p_s$, and $\oplus$ denotes the modula 2 addition. Sensors separately encode their observations using two parallel concatenated convolutional codes (i.e. turbo code). The coded sequence, $\mathbf{u}_n = \{u_n(k)\}_{k=1}^{K+M}$, consists of information bits, $\{u_n(k)\}_{k=1}^{K} = \{y_n(k)\}_{k=1}^{K}$, and parity bits $\{u_n(k)\}_{k=K+1}^{K+M}$. These parity bits are generated by the two parallel concatenated convolutional codes, and are punctured to achieve a desired rate. Figure 3 shows the block diagram of the turbo encoder. As shown in Fig. 3, the parities are interleaved before puncturing. The purpose of this interleaving is to spread the positions of punctured parities on the trellis. Each sensor transmits its codeword to the FC over an independent binary symmetric channel (BSC) with crossover probability p_c. The FC receives the data of all sensors as a matrix $\{r_n(k)\}_{k=1}^{K+M}$, $1 \le n \le N$, and estimates the source as a sequence $\{\hat{x}(k)\}_{k=1}^{K}$.

For ideal BSCs where $p_c = 0$, $\{y_n(k)\}_{k=1}^{K}$, $1 \le n \le N$, are available at the FC. In this case, it is straightforward to show that the maximum likelihood estimator is[4]:

[4]Note that when N is an even number, $\sum_{n=1}^{N} y_n(k) = \frac{N}{2}$ gives no information about $x(k)$, and the bit error probability remains 0.5. Therefore, in this case we let the estimator in (1) always reproduce a zero.

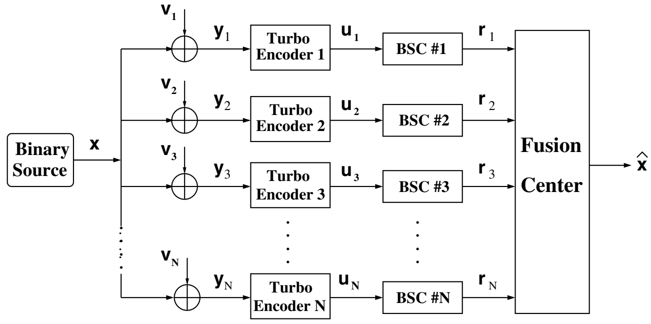

Fig. 2. Block diagram of a binary CEO problem. N sensors separately encode and transmit their observations to a fusion center through independent binary symmetric channels (BSCs).

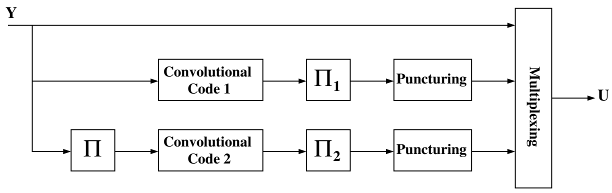

Fig. 3. Block diagram of the turbo encoder.

$$\hat{x}(k) = \begin{cases} 0 & \sum_{n=1}^{N} y_n(k) \le \frac{N}{2} \\ 1 & \text{otherwise} \end{cases} \tag{1}$$

Estimating $x(k)$ using (1) gives the bit error probability expressed by (2) (the probability that majority of sensors vote for a value that is different from the actual value of $x(k)$):

$$\Pr(\hat{x}(k) \neq x(k)) =$$
$$\begin{cases} \frac{1}{2}\binom{N}{\frac{N}{2}} p_s^{\frac{N}{2}}(1-p_s)^{\frac{N}{2}} + \sum_{n=\frac{N}{2}+1}^{N} \binom{N}{n} p_s^n (1-p_s)^{N-n} & N \text{ even} \\ \sum_{n=\frac{N+1}{2}}^{N} \binom{N}{n} p_s^n (1-p_s)^{N-n} & N \text{ odd} \end{cases} \tag{2}$$

This is the *minimum bit error probability* that the system could achieve by applying any coding scheme; i.e. regardless of which error correcting code is applied, the bit error rate cannot be reduced below this minimum.

III. Global Decoder and Extended-Serial Decoding

The simplest way to estimate the source at the FC is to separately decode the codeword transmitted by each sensor, and then use (1). We call this method as the *separate decoding* method. As an alternative approach, Howard *et al.* propose [11] to regard the correlation between $\mathbf{x}$ and $\mathbf{y}_n$'s as a *global code* and design a SISO *global decoder*. This decoder can be concatenated with the BCJR decoders of the codes applied to encode data sequences at sensors. Therefore, an iterative joint decoding becomes possible.

In this section we first model the correlation between the source and sensors observations by a graph. Then, we apply the sum-product algorithm to find the extrinsic LLRs. At the end of this section, we show how the FC applies extended-serial decoding to jointly estimate the source.

A. Global Decoder

Let $\mathbf{y}^N(k) = (y_1(k), y_2(k), .., y_N(k))$, and let $p_{X(k)}(x(k))$ represent the probability mass function of $x(k)$ (in this work we assume $p_{X(k)}(0) = p_{X(k)}(1) = \frac{1}{2}$). The correlation between $x(k)$ and $\mathbf{y}^N(k)$ can be expressed by a function $f_k(x(k), \mathbf{y}^N(k))$ as:

$$f_k(x(k), \mathbf{y}^N(k)) = \alpha p_{X(k)}(x(k)) p_s^{w_k}(1-p_s)^{N-w_k}, \quad (3)$$

where $w_k = \sum_{n=1}^{N}((x(k) \oplus y_n(k))$, and α is a coefficient that normalizes $\sum_{x(k),\mathbf{y}^N(k)} f(x(k), \mathbf{y}^N(k))$. We construct a bipartite graph with a function node f_k, $N+1$ variable nodes $x(k), \mathbf{y}^N(k)$, and $N+1$ edges connecting all variable nodes to f_k. According to [19], each variable node sends a message vector to f_k. For example, the 2-tuple message vector sent from $y_n(k)$ to f_k is shown by $\mu_{y_n(k)\to f_k}(y_n(k))$ for $y_n(k) = 0, 1$. According to sum-product update rule [19], this message is expressed as:

$$\begin{aligned}\mu_{y_n(k)\to f_k}(y_n(k)) \quad &= \quad \beta \times \mu_{c\to y_n(k)}(y_n(k)) \\ &\quad\quad \times \mu_{h_1\to y_n(k)}(y_n(k)) \\ &\quad\quad \times \mu_{h_2\to y_n(k)}(y_n(k)) \quad (4)\end{aligned}$$

where β is normalizing the summation of messages to one. $\mu_{c\to y_n(k)}(y_n(k))$, $\mu_{h_1\to y_n(k)}(y_n(k))$, and $\mu_{h_2\to y_n(k)}(y_n(k))$, represent messages that $y_n(k)$ receives from the channel, the first BCJR decoder, and the second BCJR decoder, respectively. In this case, these messages are the *a priori* probability distributions and can be calculated from the *a priori* LLRs that the global decoder receives from the channel and the two BCJR decoders. For the variable node $x(k)$, we set $\mu_{x(k)\to f_k}(x(k)) = p_{X(k)}(x(k))$. After receiving messages from all variable nodes, f_k sends a message $\mu_{f_k\to y_n(k)}(y_n(k))$ to each $y_n(k)$ ($n = 1$ to N). By applying the sum-product update rule [19], $\mu_{f_k\to y_n(k)}(y_n(k))$ is expressed as follows :

$$\begin{aligned}&\mu_{f_k\to y_n(k)}(y_n(k)) \\ &= \gamma \sum_{\sim\{y_n(k)\}} f_k(x(k), \mathbf{y}^N(k)) \times \mu_{x(k)\to f_k}(x(k)) \\ &\quad\quad\quad \times \prod_{i=1, i\neq n}^{N} \mu_{y_i(k)\to f_k}(y_i(k)) \quad (5)\end{aligned}$$

where $\sum_{\sim\{y_n(k)\}}$ denotes summation over all variable nodes excluding $y_n(k)$. The coefficient γ is applied to normalize the summation of messages to one. Finally the extrinsic LLR for each $y_n(k)$ is expressed by $\log \frac{\mu_{f_k\to y_n(k)}(0)}{\mu_{f_k\to y_n(k)}(1)}$. The global decoder repeats the same procedure for all k ($k = 1$ to K) to calculate all extrinsic LLRs and then feeds them back to the BCJR decoders.

B. Extended-Serial Decoding Structure

A decoding structure defines how extrinsic information is exchanged between the two BCJR decoders and the global decoder. An extensive study of [16] considers eight different decoding structures, and shows the superiority of an extended-serial structure among all eight structures. The extended-serial structure is generally accepted as the best method to exchange extrinsic information between three SISO modules. The structure of an extended-serial decoder is shown in Fig. 4. The global decoder runs in serial with the BCJR decoders; and each decoder passes its extrinsic information to the other two decoders for their next decoding iteration. After a definite number of iterations, the fusion center estimates the value of $x(k)$ as follows:

First the FC calculates vectors

$$\widetilde{\mu}_{y_n(k)}(y_n(k)) = \mu_{f_k\to y_n(k)}(y_n(k)) \times \mu_{y_n(k)\to f_k}(y_n(k)), \quad (6)$$

for $y_n(k) = 0, 1$. This is equivalent to calculating the sum of all most recent extrinsic LLRs [20] (received from both BCJR decoders and the global decoder). Then, the FC calculates the marginal function

$$\begin{aligned}\widetilde{\mu}(x(k)) \quad = \quad &\sum_{\sim\{x(k)\}} f_k(x(k), \mathbf{y}^N(k)) \times p_{X(k)}(x(k)) \\ &\times \prod_{n=1}^{N} \widetilde{\mu}_{y_n(k)}(y_n(k)) \quad (7)\end{aligned}$$

Finally the value of $x(k)$ is estimated as

$$\hat{x}(k) = \begin{cases} 0 & \widetilde{\mu}(0) \geq \widetilde{\mu}(1) \\ 1 & \text{otherwise} \end{cases} \quad (8)$$

Note that $x(k)$ can alternatively be estimated by looking at *a posteriori* LLRs of the last active decoder, making a binary decision on $y_n(k)$'s, and then deciding based on the majority of the outputs[5]. However, we found by experiments that estimating $x(k)$ from (8) will lead to lower bit error probabilities.

IV. Simulation Results

We implement the turbo encoder according to Fig. 3. Both convolutional codes have generator polynomials $\frac{1+D^2}{1+D+D^2}$; the message block length is $K = 1024$ bits; and the number of sensors is $N = 4$. Interleavers $\mathbf{\Pi}$, $\mathbf{\Pi}_1$, and $\mathbf{\Pi}_2$ are generated as three independent random permutations and are fixed for turbo encoders of all 4 sensors. To adjust the code rate to a given value R, $\frac{3K}{2} - \frac{K}{2R}$ parities are punctured for each convolutional code. This means that each convolutional code transmits $\frac{K}{2R} - \frac{K}{2}$ of its parities. Therefore, the length of the codeword is $K + 2 \times (\frac{K}{2R} - \frac{K}{2}) = \frac{K}{R}$, and the code rate

[5]Note that this is not the same as separate decoding, since no global decoder is involved in the separate decoding.

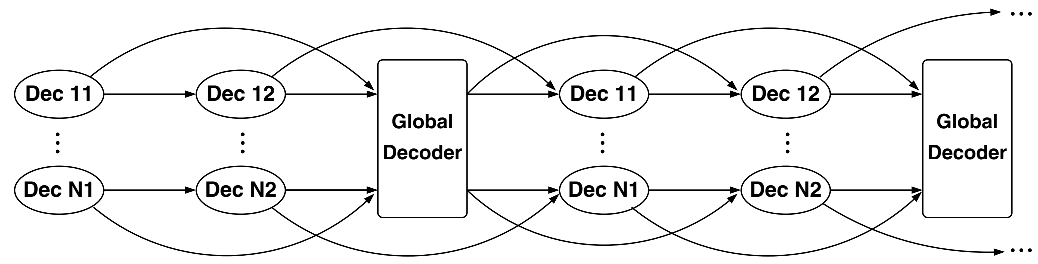

Fig. 4. The structure of extended-serial decoding for turbo-coded sensor networks. Message passing among all SISO decoders, i.e., the global decoder and the BCJR decoders of all N sensors, is illustrated. $\mathrm{Dec}\,n1$ and $\mathrm{Dec}\,n2$ denote the BCJR decoders corresponding to sensor n ($n = 1, \ldots, N$).

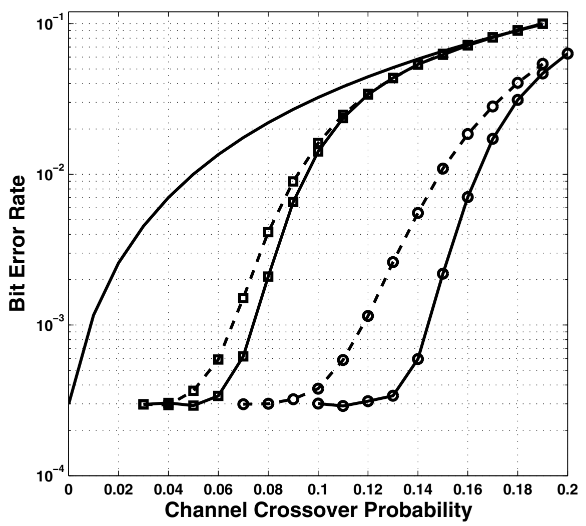

Fig. 5. Bit error rate achieved by $Rate = \frac{1}{2}$ turbo codes for separate decoding (Squares) and extended-serial decoding (Circles), after 2 iterations (Dashes) and 4 iterations (Solid). The unmarked solid line shows the bit error rate of uncoded transmission for $p_s = 0.01$, and $N = 4$.

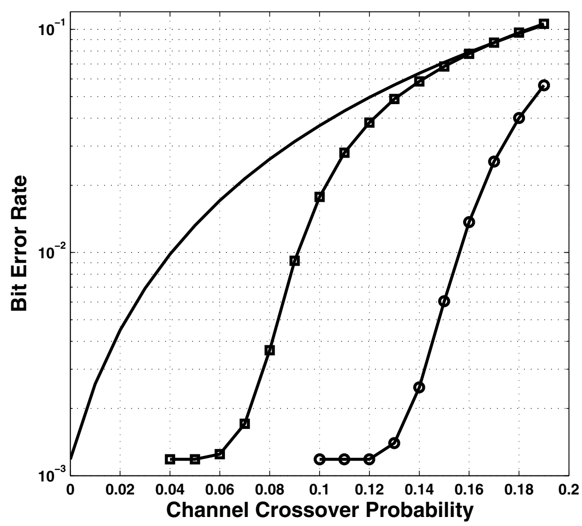

Fig. 6. Bit error rate achieved by $Rate = \frac{1}{2}$ turbo codes for separate decoding (Squares) and extended-serial decoding (Circles) after 4 iterations. The unmarked solid line shows the bit error rate of uncoded transmission for $p_s = 0.02$, and $N = 4$.

is R. The bit error rate for uncoded transmission is found if we replace p_s in (2) by $p_c(1-p_s) + p_s(1-p_c)$ (the total crossover probability between the source and the FC). For comparison, bit error rate curves for uncoded transmission are also provided in all figures.

Figure 5 shows the bit error rate achieved by rate $\frac{1}{2}$ turbo codes after 2 and 4 iterations. The crossover probability between source and sensors observations is $p_s = 0.01$. From (2), the minimum achievable bit error probability is 3.0×10^{-4}. By performing 2 decoding iterations, separate decoding achieves this minimum bit error rate for channel crossover probabilities of $p_c \leq 0.04$, whereas extended-serial decoding reaches this minimum for all $p_c \leq 0.09$. By increasing the number of iterations to 4, these values increase to $p_c \leq 0.05$ and $p_c \leq 0.11$, respectively. For 2 iterations and $p_c = 0.16$, separate decoding does not offer a bit error rate lower than the one achieved by uncoded transmission (7.2×10^{-2}). However, extended-serial decoding achieves a bit error probability of 1.8×10^{-2}, i.e. it offers a four-time decrease of bit error probability compared with the uncoded transmission. As another comparison, assume that a bit error rate of 10^{-3} is desired and the FC performs 4 decoding iterations. By performing 4 iterations, separate decoding is able to achieve the desired bit error rate if $p_c \leq 0.08$, whereas extended-serial decoding achieves this bit error rate for $p_c \leq 0.14$. In other words, extended-serial decoding achieves the desired bit error rate for channels that produce up to $\frac{0.14}{0.08} = 1.75$ times more errors.

Figure 6 shows the bit error rate curves for 4 iterations, when p_s is increased to 0.02. In this case, the minimum achievable bit error rate is increased to 1.2×10^{-3}. The separate decoding achieves this minimum for $p_c \leq 0.06$, whereas extended-serial decoding reaches this minimum for all $p_c \leq 0.12$. Figure 7 shows the bit error rate curves for rate $\frac{4}{7}$ codes and $p_s = 0.01$. If we assume that a bit error rate of 10^{-3} is desired, separate decoding and extended-serial decoding achieve this desired bit error rate for $p_c \leq 0.06$ and $p_c \leq 0.12$, respectively. This means that extended-serial decoding is able to achieve this desired bit error rate for channels that produce up to 2.0 times more errors.

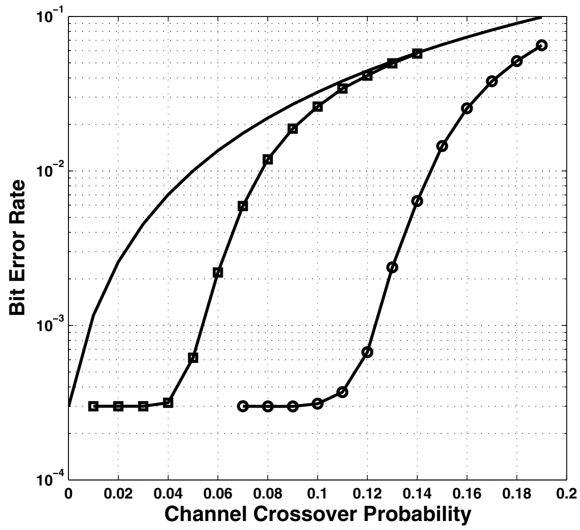

Fig. 7. Bit error rate achieved by $Rate = \frac{4}{7}$ turbo codes for separate decoding (Squares) and extended-serial decoding (Circles) after 4 iterations. The unmarked solid line shows the bit error rate of uncoded transmission for $p_s = 0.01$, and $N = 4$.

V. Conclusion

We considered a specific type of data gathering sensor networks that can be modeled by the binary CEO problem. We applied turbo codes to encode sensors observations and transmit them to the FC. According to the literature, the FC is able to exploit the correlation between sensors observations and design a SISO global decoder. We applied the sum-product algorithm to derive the update rules for the global decoder. Then, we considered an extended-serial concatenation of this global decoder with the two BCJR decoders of turbo code, to jointly estimate the source at the FC. We compared the achieved bit error rate with the one achieved by a separate decoding scheme. Our simulations showed that the extended-serial decoding performs significantly better than the separate decoding.

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