ITERATIVE JOINT DECODING FOR SENSOR NETWORKS WITH BINARY CEO MODEL

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ABSTRACT

An iterative joint decoding algorithm for data gathering wireless sensor networks is proposed in [1] where the correlation between sensors' data is considered as a *global code* and iterative decoding is performed by concatenating the *global decoder* with the decoder of error correcting code applied to encode sensors observations. We apply this algorithm for sensor networks with binary CEO model where sensors observe different noisy versions of a single source, located away from sensors. This calls for employing more powerful error correcting codes, therefore we apply convolutional codes (Hamming codes and single parity check codes are applied in [1]). We use the concept of iterative horizontal-vertical decoding for concatenated block codes to formulate the update rules for L-values for the considered binary CEO model. Our simulations confirm that the iterative joint decoding scheme substantially decreases the bit error rate compared with the separate decoding scheme, and reaches the minimum achievable distortion for channels with significantly higher noise levels.

Index Terms— CEO problem, sensor networks, iterative decoding, convolutional code.

1. INTRODUCTION

Wireless sensor networks (WSNs) are widely recognized as one of the most promising emerging technologies in the field of communications and information technology. New advances in hardware and wireless network technologies as well as in low power electronics have reduced the cost, size, and power of micro-sensors [2]. This enables us to employ distributed wireless sensing in a wide range of applications [3], including environmental monitoring, rescue operations, monitoring seismic propagation in buildings, health care and medical applications.

We focus on the application of WSNs in data gathering. This type of WSN has the same model as the Chief Executive Officer (CEO) problem [4], where a CEO is interested in a source that cannot be observed directly. N agents make independent noisy observations of the source, separately encode their observations, and then transmit through rate-constrained channels to a single fusion center (FC) for further processing. The scenario is shown in Fig. 1. The CEO problem is considered for discrete case in [4] and for the quadratic Gaussian case in [5, 6, 7, 8, 9].

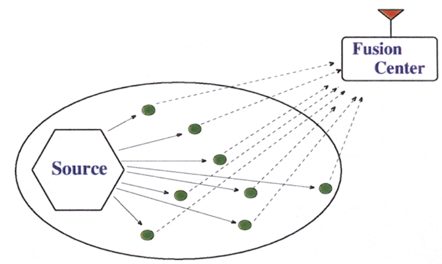

Fig. 1. A data gathering wireless sensor network with $N = 8$ sensors.

Each sensor node in a WSN has a limited power supply which is not rechargeable. Also, it is not easy to service a large number of sensor nodes in remote, possibly inaccessible locations. Thus, the key challenge is conserving the energy of the distributed wireless sensor nodes and maximizing their lifetime. Accordingly, it is desired to exploit the correlation between sensors' data in order to reduce the transmission rate. In fact, since the bit-rate directly impacts power consumption at a sensor node, by eliminating the data redundancy and reducing the communication load, we can manage the energy resources in an optimal manner. Since the sensors are not allowed to communicate, the only way to exploit the correlation is to design the decoder (located in the FC) to take this correlation into account. Such decoder could alternatively achieve the same error probability for a smaller transmission rate, and save energy.

In [1] an iterative joint decoding algorithm is proposed for data gathering WSNs. The algorithm regards the correla-

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tion between sensors' data as a *global code*. The authors in [1] then show how to design a decoder for this global code. This decoder is concatenated with the decoder of the error correcting code that is applied to encode each sensor's data. Extrinsic information is passed between decoders, and decoding continues for a definite number of iterations. In [1] such iterative decoder is applied to data gathering Gaussian sensor networks where it is shown that this decoding algorithm performs substantially better than a separate decoding scheme in sense of achieving smaller mean square distortion. The performance is also compared with the performance of a MAP decoder and is shown to be slightly worse. For MAP decoding, the correlation is considered in the form of joint probability distributions. A similar algorithm is proposed in a pioneer work of [10], where low-density generator matrix (LDGM) codes are applied to separately encode sensors' observations. The receiver uses a graphical model that considers the whole system as a single graph, and runs belief propagation (BP) algorithm on this graph to decode the source.Other similar works appear in [11, 12].

We apply the iterative joint decoding algorithm to binary sensor networks modeled by a binary CEO problem. In contrast to [1] that considers sensors observations as different correlated sources (e.g. temperature and moisture of a room that are naturally correlated), we consider all sensors observations to be originated from a single (binary) source, but corrupted by independent noises. We use the concept of iterative horizontal-vertical decoding [13] to formulate the update rules for the *L*-values for the *global code*. Existence of observation noises calls for applying more powerful error correcting codes at sensor nodes. The reason is that if the sensor observation is decoded by a definite decoding error probability, this error probability increases when the source is being estimated from this decoded observation. Therefore, in contrast to [1] that considers very simple codes, i.e. $(7, 4)$ Hamming code and $(4, 3)$ single parity check code, we apply convolutional codes to further improve the error correcting capabilities. Since convolutional codes could be implemented by finite state machines and received sequence can be encoded and transmitted sequentially with no need for storage space, convolutional codes are ideal choices for small sensor nodes.

Section 2 presents the system model. In Section 3 we formulate the update rules for *L*-values for the iterative joint decoder. Simulation results and discussions are presented in Section 4. Section 5 concludes the paper.

2. SYSTEM MODEL

Figure 2 shows the binary CEO model for a WSN. The unbiased binary source sequence $\mathbf{x} = \{x(k)\}_{k=1}^{K}$, $\Pr(x(k) = 0) = \Pr(x(k) = 1) = 0.5$, with message block length of K, is monitored by N sensors. Sensor number n receives a noisy observation of the source sequence as $\mathbf{y}_n = \{y(n,k)\}_{k=1}^{K}$,where $y(n,k) = x(n,k) \oplus v(n,k)$, where $\oplus$ denotes the

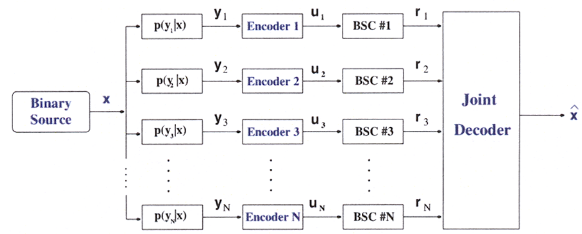

Fig. 2. A sensor network with binary CEO model.

modulus 2 summation and $v(n,k)$ is a binary i.i.d. random variable with $Pr(v(n,k) = 1) = p_s$. Each sensor encodes its data by adding M parities (systematic coding), and transmits its coded sequence $\mathbf{u}_n = \{u(n,k)\}_{k=1}^{K+M}$ to a fusion center. The fusion center receives the data of all sensors as a matrix $\{r(n,k)\}_{k=1}^{K+M}$, $1 \leq n \leq N$ and estimates the source as a sequence $\{\hat{x}(k)\}_{k=1}^{K}$. We assume that each sensor is connected to the fusion center by a binary symmetric channel with crossover probability p_c=$\Pr(r(n,k) \neq u(n,k))$. The system model is illustrated in Fig. 2. At this point the decoder has N pieces of information which are all binary sequences. The decoder must decide how to fuse these N sequences together. One solution is to separately decode each received sequence $\mathbf{r}_i = \{r(i,k)\}_{k=1}^{K+M}$ and then decide based on the majority of the outputs (voting). Another way is to apply iterative joint decoding (Section 3) to softly fuse the N pieces of information.

For ideal channels where $p_c = 0$, i.e., channels with capacity of 1 bits per channel use, $\{u(n,k)\}_{k=1}^{K+M}$, $1 \leq n \leq N$ are available at the fusion center. The optimal estimate in this ideal case reduces to a voting problem which is expressed as follows:

$$\hat{x}(k) = \begin{cases} 0 & \sum_{n=1}^{N} y(n,k) \leq \frac{N}{2} \\ 1 & \text{otherwise} \end{cases} \tag{1}$$

Estimating $x(k)$ using (1) gives a bit error probability of (2) which is the probability that majority number of sensors vote for a value that is different from the actual value of $x(k)$. Theoretically speaking, this is the minimum bit error probability that can be found when the decoder has direct access to the observations of the sensors. In fact, this minimum can be achieved if there exists a channel code with rate $\frac{K}{K+M}$ at each sensor such that it can asymptotically achieve zero error probability for the BSC with crossover probability p_c. However, if the required power to implement the capacity-achieving channel code (with probably high encoding and/or decoding complexity) is greater than the power saved due to its coding gain, it is not an energy-efficient option for WSNs [14, 15, 1]. In this work, we consider short block length systematic recursive convolutional codes and evaluate their performance in binary CEO sensor networks, by applying the iterative joint decod-

Table 1. The schematic of receiving and encoding data by sensors

$x(1)$	...	$x(K)$		
$u(1,1)$	...	$u(1,K)$	...	$u(1,K+M)$
$u(2,1)$	...	$u(2,K)$	...	$u(2,K+M)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$u(N,1)$	...	$u(N,K)$	...	$u(N,K+M)$

ing algorithm of [1] at the fusion center.

$$\Pr(\hat{x}(k) \neq x(k)) =$$

$$\begin{cases} \frac{1}{2}\binom{N}{\frac{N}{2}}p_s^{\frac{N}{2}}(1-p_s)^{\frac{N}{2}} + \sum_{n=\frac{N}{2}+1}^{N}\binom{N}{n}p_s^n(1-p_s)^{N-n} & N \text{ even} \\ \sum_{n=\frac{N+1}{2}}^{N}\binom{N}{n}p_s^n(1-p_s)^{N-n} & N \text{ odd} \end{cases} \tag{2}$$

3. IMPLEMENTATION OF ITERATIVE JOINT DECODING ALGORITHM FOR BINARY CEO MODEL

Table 1 shows the schematic of receiving source sequence $\{x(k)\}_{k=1}^{K}$ by each sensor and encoding it to a sequence $\{u(n,k)\}_{k=1}^{K+M}$. The vectors $\{r(n,k)\}_{k=1}^{K+M}$ are received at the decoder. The decoder sets the log-likelihood values and performs an iterative decoding by running alternative horizontal and vertical decodings [13]. Table 2 shows the L-values received from the BSCs. Given a crossover probability $p_c < 0.5$, the reliability value of the channel is $L_c = log\frac{1-p_c}{p_c}$ [13]. Therefore, the L-values for the received vectors are set to $\{L_c.r(n,k)\}_{k=1}^{K+M}, 1 \leq n \leq N$. The L-values for source bits $\{x(k)\}_{k=1}^{K}$ are set to zero, indicating that no information is available for these bits. Also, the extrinsic L-values received from vertical decoding are initialized as $\{L_e^|(n,k)\}_{k=1}^{K} = 0$, for all n. After setting the L-values, horizontal and vertical decodings are performed sequentially as follows:

Horizontal Decoding: The BCJR algorithm [16] is performed on the trellis of each convolutional code ($n = 1, 2, ..., N$) and the vector of *a posteriori* L-values $\{L^-(n,k)\}_{k=1}^{K}$ for each n is obtained. Then the extrinsic L-values are calculated as

$$L_e^-(n,k) = L^-(n,k) - L_c.r(n,k) - L_e^|(n,k). \tag{3}$$

The vector of L-values $\{L_e^-(n,k) + L_c.r(n,k)\}_{k=1}^{K}$ is used as *a priori* values for vertical decoding.

Vertical Decoding: First we calculate initial probabilities using *a priori* L-values received from horizontal decoding:

$$\Pr(y(n,k)=0) = \frac{exp(L_e^-(n,k) + L_c.r(n,k))}{1 + exp(L_e^-(n,k) + L_c.r(n,k))},$$

Table 2. The L-values received from BSCs

0	...	0		
$L_c.r(1,1)$	...	$L_c.r(1,K)$	...	$L_c.r(1,K+M)$
$L_c.r(2,1)$	...	$L_c.r(2,K)$	...	$L_c.r(2,K+M)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$L_c.r(N,1)$	...	$L_c.r(N,K)$	...	$L_c.r(N,K+M)$

and

$$\Pr(y(n,k)=1) = \frac{1}{1 + exp(L_e^-(n,k) + L_c.r(n,k))}.$$

Therefore, if $f(y(n,k))$ represents the probability mass function of $y(n,k)$, then:

$$f(y(n,k)) = \begin{cases} \frac{exp(L_e^-(n,k)+L_c r(n,k))}{1+exp(L_e^-(n,k)+L_c r(n,k))} & y(n,k)=0 \\ \frac{1}{1+exp(L_e^-(n,k)+L_c r(n,k))} & y(n,k)=1 \end{cases} \tag{4}$$

Then we estimate a joint probability mass function of $f(x(k), y(1,k), ..., y(N,k))$ as:

$$f(x(k), y(1,k), ..., y(N,k)) =$$

$$\frac{1}{2}\alpha p_s^w(1-p_s)^{N-w}\prod_{n=1}^{N} f(y(n,k)) \tag{5}$$

where $w = \sum_{n=1}^{N}((x(k) \oplus y(n,k))$. The coefficient $\frac{1}{2}$ represents the probability of $x(k)$ and α is a coefficient that normalizes the following summation to one:

$$\sum_{x(k)=0}^{1}\sum_{y(1,k)=0}^{1}\cdots\sum_{y(N,k)=0}^{1} f(x(k), y(1,k), ..., y(N,k)) = 1. \tag{6}$$

Then for each n we calculate the marginal probability mass function:

$$\tilde{f}(y(n,k)) = \beta \times \sum_{x(k)=0}^{1}\sum_{y(1,k)=0}^{1}\cdots\sum_{y(n-1,k)=0}^{1}$$

$$\sum_{y(n+1,k)=0}^{1}\cdots\sum_{y(N,k)=0}^{1} f(x(k), y(1,k), ..., y(N,k)), \tag{7}$$

for $y(n,k) = 0, 1$, respectively. Again, coefficient β is used for normalization. Finally, the *extrinsic* L-value is expressed by $L_e^|(n,k) = \log\frac{\tilde{f}(0)}{\tilde{f}(1)}$. These extrinsic L-values are added to $L_c.r(n,k)$ to initialize the BCJR algorithm for the subsequent horizontal decoding. One decoding iteration includes the complete decoding of both the horizontal and vertical decoding. If a binary decision on $y(n,k)$ is required, such decision can be made by calculating the *a posteriori* L-values as

$$L^|(n,k) = L_c.r(n,k) + L_e^|(n,k) + L_e^-(n,k). \tag{8}$$

Final Estimation: After performing a definite number of iterations (e.g. 5 iterations), we calculate the marginal probabilities for $x(k)$ as

$$\Pr(x(k) = b) = \sum_{y(1,k)=0}^{1} \dots \sum_{y(N,k)=0}^{1} f(b, y(1,k), ..., y(N,k)), \tag{9}$$

where $b = 0$, 1, and function $f(.)$ is expressed by (5). Then we estimate $x(k)$ as

$$\hat{x}(k) = \begin{cases} 0 & \text{if } \Pr(x(k) = 0) \geq \Pr(x(k) = 1) \\ 1 & \text{otherwise} \end{cases} \tag{10}$$

4. SIMULATION RESULTS

In all simulations we consider systematic recursive convolutional codes with generator polynomials $(1, \frac{1+D^2}{1+D+D^2})$. For the case of the iterative joint decoding algorithm, the number of iterations is fixed to 4. The source block length is fixed to $K = 1000$ bits. The crossover probability between source and sensors is fixed to $p_s = 0.01$. Fig. 3 shows the bit error rate of the system using two different decoding methods. The first method separately decodes each vector $\{r(n,k)\}_{k=1}^{K}$ for each n, and then obtains the estimate $\hat{x}(k)$ from (1). The second method applies the iterative joint decoding algorithm. It is observed that iterative joint decoding performs substantially better than the separate decoding. The minimum achievable bit error probability is found from (2) to be 3.0×10^{-4}. It is observed from Fig. 3 that the separate decoding approaches this minimum bit error probability for channels with $p_c \leq 0.01$. However, the joint decoding method approaches the minimum bit error probability for all BSCs with $p_c \leq 0.05$. For comparison, the bit error probability of an uncoded scheme is also shown in Fig. 3. The uncoded transmission over the BSC results in a total crossover probability of $p_s(1-p_c) + p_c(1-p_s)$ between $x(k)$ and each received bit $r(n,k)$. The bit error probability of this transmission could be found after replacing p_s in (2) by $p_s(1-p_c) + p_c(1-p_s)$. While the uncoded transmission achieves the minimum error probability of (2) only for the ideal channel (i.e. $p_c = 0$, not shown in Fig. 3), we observe that coded transmissions could approach this minimum error probability for values of $p_c > 0$.

Fig. 4 shows the bit error rate achieved by 4 iterations of joint decoding algorithm, while using $N = 4$, and $N = 6$ sensors. As expected, applying $N = 6$ sensors decreases the bit error probability. At $p_c = 0.05$ both schemes approach their minimum achievable bit error probabilities that are 3.0×10^{-4} and 9.9×10^{-6}, respectively (found from (2)). It is possible to increase the coding rate at the expense of increasing bit error rate in order to provide better spectral efficiency. More importantly, increasing the coding rate reduces the transmission power consumption in sensor nodes and increases the network lifetime. Figure 5 studies the effect of increasing the coding

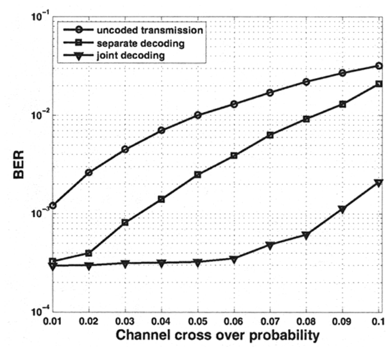

Fig. 3. Bit error rate of a system with $N = 4$ sensors and $(2000, 1000)$ convolutional codes, using separate decoding (Squares) and joint decoding (Triangles).

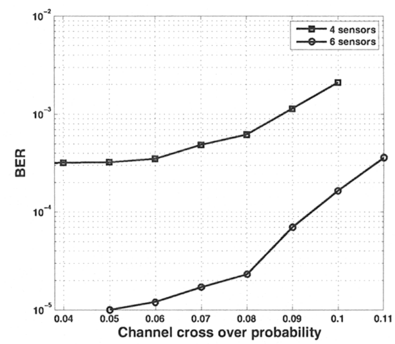

Fig. 4. Bit error rate of systems with $N = 4$ (Squares) and $N = 6$ (Circles) sensors. $(2000, 1000)$ convolutional codes are applied.

rate for a system with $N = 6$ sensors. From $M = 1000$ generated parities, 200 of them are punctured to increase the rate from $\frac{1}{2}$ to $\frac{5}{9}$. The positions of punctured parities are selected randomly and are fixed after selection. These position are know at the decoder, and the decoder sets their corresponding L-values to zero. It is observed that error correcting capability of the code is affected by puncturing. The non punctured rate $\frac{1}{2}$ code achieves bit error probability of 10^{-4} for a channel with $p_c = 0.095$, whereas the punctured code of rate $\frac{5}{9}$ reaches the same bit error probability only after decreasing p_c to 0.075. At $p_c = 0.06$, the non-punctured code has nearly reached the minimum error probability (that is 9.9×10^{-6}) but the punctured code gives a higher bit error rate of 4.4×10^{-5}.

5. CONCLUSION

We considered a binary wireless sensor network, modeled by the binary CEO problem. We applied the iterative decoding algorithm presented in [1] which gives us the ability to softly

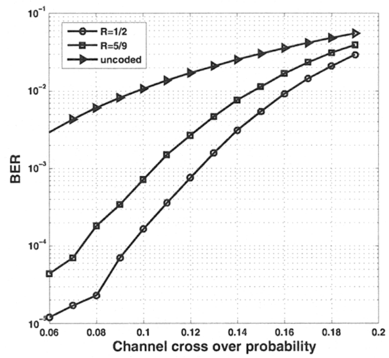

Fig. 5. Bit error rate of a system with $N = 6$ sensors when a $(2000, 1000)$ convolutional code (Circles) and a $(1800, 1000)$ convolutional code (Squares) are applied. The second code is generated by puncturing 200 parities from the first code.

fuse binary sequences received from different sensors. In contrast to [1], in our model all sensors observe noisy versions of the same source. The already existing noise before transmission, calls for more powerful channel coding. Therefore, we applied convolutional codes of message block length 1000. Our simulation results for different number of sensors and different transmission rates confirmed that the iterative joint decoding scheme significantly improves the bit error probability of binary wireless sensor networks compared with the separate decoding scheme. Also, the iterative joint decoder reaches the minimum achievable bit error rate for BSCs with significantly higher noise levels.

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