Analysis of unidirectional grating-assisted co-directional couplers by transfer matrix method

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Abstract: The unidirectional grating-assisted co-directional coupler is the basis for a new generation of photonic devices. We analyze this device by the transfer matrix method, which leads to a simple explanation of its unidirectional coupling properties.

Keywords: photonics, gratings, couplers

Introduction

Power transfer between waveguide modes can be achieved using a refractive index grating of period Λ in the region where the mode fields overlap [1]. When the grating vector K_g is equal to the difference in modal propagation constants,

$$\beta_m - \beta_n = K_g = 2\pi / \Lambda, \qquad [1]$$

complete power transfer can be obtained. This process is periodic over the length of the grating; in the spatial frequency domain, a pure index grating exhibits dual sidebands at $\pm K_g$, equalizing the coupling strength between modes *m* and *n*. These structures are known as grating-assisted couplers (GACs). In general, GACs can couple power between forward- and backward-propagating modes within the same waveguide (Bragg grating), or between forward-propagating modes in asynchronous waveguides (long-period grating).

Recently, there has been significant interest in a variation of the GAC in which single-sideband index modulation results in unidirectional power transfer [2]. This structure shows great promise in optical routing as a lossless add multiplexer [3] and as an optical memory cell [4,5]. Despite the vast potential for these devices, very little progress has been made in their design and analysis. The earliest work used resonance mode expansion to derive the unidirectional coupling properties [6], and all subsequent analyses have used coupled mode theory (CMT) [3,7,8]. CMT is known to be inaccurate for actual design work, particularly in the case of large index perturbation, or when the teeth of the grating have surfaces oriented perpendicular to the guided modes' polarization [9]. In this work, we analyze an ideal unidirectional GAC by the well-known transfer matrix method (TMM). The single-period transfer matrix is derived, and from this matrix it is easily seen that the power transfer is indeed unidirectional. A simple expression is found for the number of grating periods required for power equalization in each mode, and it is shown that there is a fundamental limitation to the minimum field "leakage" in the non-coupling direction. Note that in this work we consider only the unidirectional grating-assisted co-directional coupler (U-GACC) utilizing a long-period grating, although the methods discussed here are equally applicable to Bragg gratings.

2. Description of the U-GACC

The U-GACC considered in this work consists of two parallel, asynchronous, single-mode waveguides (Fig. 1). The combined waveguides support two orthogonal supermodes. For the large asynchronicity required for relatively short grating periods, the supermodes closely resemble the modes of each isolated waveguide. Therefore, for the purposes of this work, we consider the power in waveguide 1 (2) to be equivalent to the power in supermode 1 (2). The two supermodes are coupled by a longitudinal perturbation in the complex refractive index. In an ideal U-GACC, the real and imaginary parts of the index perturbation are equal in magnitude and separated by $\pi/2$ in phase. A more detailed conceptual description of the U-GACC can be found in Refs. [2-4].

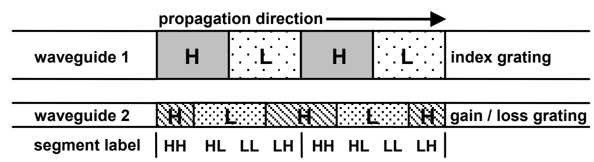

Figure 1 : Generalized structure of a U-GACC (two periods shown). In this embodiment, the real index grating is in waveguide 1 and the gain/loss grating is in waveguide 2

It is clear from Fig. 1 that the grating consists of a repetition of four discrete segments (as opposed to two for a real grating). These segments are given index *RI* (*R*, *I* = "*H*," "*L*") to indicate whether the real (*R*) and imaginary (*I*) parts of the index perturbation are higher or lower than the average index, respectively.

3. Transfer matrix method analysis

The TMM has been used extensively in the analysis of photonic devices exhibiting longitudinal periodicity [9], and is especially beneficial when the index profile is piecewise continuous in the propagation direction – precisely the situation that is encountered with most long-period gratings. As the TMM uses actual mode fields in each region of the corrugation and not averaged mode fields as with CMT, large index perturbations are permissible.

The transfer matrix of a complete period consists of four transfer matrices representing the grating planes, separated by four matrices representing free propagation of each mode. The transfer matrix for propagation from section *RI* to section *R'I'* is given by

$$\mathbf{T}^{(R'I')RI} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix}, \qquad [2]$$

where

$$t_{mn}^{(R'I')RI} = \frac{1}{4}\iint \left[\mathbf{E}_m^{R'I'} \times \mathbf{H}_n^{RI} + \mathbf{E}_n^{R'I'} \times \mathbf{H}_m^{RI}\right] \cdot \hat{z} da \,. \qquad [3]$$

Note that we are dealing with complex modes which are orthonormal in a self-reactance sense, as evident in Eq. [3]. After lengthy calculations, it can be shown that the transfer matrices for real and imaginary grating planes of equal strength δ are

$$\mathbf{T}^{(HH)LH} = \begin{pmatrix} 1-\frac{\delta^2}{2} & -\delta \\ \delta & 1-\frac{\delta^2}{2} \end{pmatrix}, \mathbf{T}^{(LL)HL} = \left(\mathbf{T}^{(HH)LH}\right)^{\mathrm{T}}$$
$$\mathbf{T}^{(LH)LL} = \begin{pmatrix} 1+\frac{\delta^2}{2} & -i\delta \\ i\delta & 1+\frac{\delta^2}{2} \end{pmatrix}, \mathbf{T}^{(HL)HH} = \left(\mathbf{T}^{(LH)LL}\right)^{\mathrm{T}}. \quad [4]$$

The propagation matrices for an ideal U-GACC are

$$\mathbf{P}^{HH} = \begin{pmatrix} \frac{(1-i)g_1^H}{\sqrt{2}} & 0 \\ 0 & \frac{(1+i)g_2^H}{\sqrt{2}} \end{pmatrix}, \mathbf{P}^{HL} = \begin{pmatrix} \frac{(1-i)}{\sqrt{2}g_1^H} & 0 \\ 0 & \frac{(1+i)}{\sqrt{2}g_2^H} \end{pmatrix},$$
$$\mathbf{P}^{LH} = \begin{pmatrix} \frac{(1-i)g_1^L}{\sqrt{2}} & 0 \\ 0 & \frac{(1+i)g_2^L}{\sqrt{2}} \end{pmatrix}, \mathbf{P}^{LL} = \begin{pmatrix} \frac{(1-i)}{\sqrt{2}g_1^L} & 0 \\ 0 & \frac{(1+i)}{\sqrt{2}g_2^L} \end{pmatrix}, \quad [5]$$

where $g_m^{H(L)}$ is the gain experienced by mode m upon traversing the amplifying segment with high (low) real index, and $1/g_m^{H(L)}$ is the (matched) loss in the corresponding lossy segment. Note that the two modes de-phase by exactly $\pi/2$ in each segment, and that a non-significant overall phase factor has been omitted.
For the U-GACC oriented as in Fig. 1, the transfer matrix for a full period is

$$\mathbf{T} = \mathbf{T}^{(HH)LH}\mathbf{P}^{LH}\mathbf{T}^{(LH)LL}\mathbf{P}^{LL}\mathbf{T}^{(LL)HL}\mathbf{P}^{HL}\mathbf{T}^{(HL)HH}\mathbf{P}^{HH}$$
$$= \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \quad [6]$$

with (to 2nd degree in δ)

$$T_{11} = 1+\delta^2\left(\frac{g_2^L}{g_1^L} + 2\frac{g_1^H}{g_2^H} - \frac{g_1^L}{g_2^L} + \frac{g_1^L g_1^H}{g_2^L g_2^H} - 1\right),$$
$$T_{12} = -\delta\left(\frac{g_2^H}{g_1^H} + \frac{g_1^L}{g_2^L} + 2\right), T_{21} = \delta\left(\frac{g_1^H}{g_2^H} + \frac{g_2^L}{g_1^L} - 2\right), \quad [7]$$
$$T_{22} = 1-\delta^2\left(\frac{g_1^L}{g_2^L} + 2\frac{g_2^H}{g_1^H} - \frac{g_2^L}{g_1^L} + \frac{g_2^L g_2^H}{g_1^L g_1^H} + 1\right).$$

In all practical cases, $|g_m^{H(L)}|$ very slightly exceeds unity, and thus $|T_{12}| >> |T_{21}|$. This strong asymmetry in **T** is the basis of the unidirectional power transfer in the U-GACC. In the limiting case where all of the gain ratios in Eq. [7] are equal to unity, the first-order evaluations of the fields after one period are

$$\mathbf{E}_{\mathrm{out}} = \mathbf{T}\mathbf{E}_{\mathrm{in}} = \begin{pmatrix} 1 & -4\delta \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad [8]$$

for input into the first mode, and

$$\mathbf{E}_{\mathrm{out}} = \mathbf{T}\mathbf{E}_{\mathrm{in}} = \begin{pmatrix} 1 & -4\delta \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -4\delta \\ 1 \end{pmatrix} \quad [9]$$

for input into the second mode. Equations [8]-[9] indicate that there is no coupling from mode 1 to mode 2, but that when the input is in mode 2, the field magnitudes equalize after $1/(4\delta)$ periods. When no assumption is made on the gain ratios, T_{21} is non-zero, and it can be seen from Eqs. [7]-[8] that a U-GACC designed for field equalization will, with input into the first mode, exhibit a field "leakage"

$$\text{leakage} \approx \frac{1}{4}\left(\frac{g_1^H}{g_2^H} + \frac{g_2^L}{g_1^L} - 2\right). \quad [10]$$

4. Conclusion

The transfer matrix method has been used to analyze a unidirectional grating-assisted co-directional coupler. In addition to providing a clear explanation of the unidirectional coupling property of the device, simple expressions have been derived for the number of periods required for field equalization in the waveguides, as well as the resulting field leakage.

5. Acknowledgment

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6. References

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