Simultaneous nonlinearity suppression and wide-band dispersion compensation using optical phase conjugation

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Abstract: Optical phase conjugation is demonstrated to enable simultaneous wide-band compensation of the residual dispersion and the fiber nonlinearities in dispersion-managed fiber transmission lines employing slope-compensating fibers. When the dispersion slope of transmission fibers is equalized by slope-compensating fibers, the residual dispersion and the slope of dispersion slope are compensated by middle-span optical phase conjugation. More importantly, fiber nonlinearity may be largely suppressed by arranging the fibers into conjugate pairs about the phase conjugator, where the two fibers of each pair are in scaled translational symmetry. The translational symmetry is responsible for cancelling optical nonlinearities of the two fibers up to the first-order perturbation, then a mirror-symmetric ordering of the fiber pairs about the conjugator linearizes a long transmission line effectively.

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1. Introduction

Group-velocity dispersion and optical nonlinearity are the major limiting factors in high-speed long-distance fiber-optic transmissions [1, 2]. Dispersion-compensating fibers (DCFs) have been developed to offset the dispersion effects of transmission fibers over a wide frequency band. The most advanced DCFs are even capable of slope-matching compensation, namely, compensating the dispersion and the dispersion slope of the transmission fiber simultaneously [3, 4]. Nevertheless, DCFs could hardly be designed and fabricated to match exactly the dispersion and the slope of transmission fibers simultaneously. In general, it is difficult to perfectly compensate the fiber dispersion across a wide frequency band. There are always residual dispersion and higher order derivatives, even using the best slope-matching DCFs [5, 6, 7]. The significance of the residual dispersions increases as the total signal bandwidth becomes wider [8]. It has been proposed for some time that optical phase conjugation (OPC) may be employed in the middle of a transmission line to equalize the dispersion effect of the transmission fibers [9]. Furthermore, theoretical and experimental studies have proved the feasibility of using OPC to compensate the fiber nonlinearities, at least partially [10, 11, 12]. In the past, the application of OPC has been limited by the lack of performing conjugators that require low pump powers, operate over wide bandwidths, and suffer low penalties. Such technical difficulties and the inability of compensating the dispersion slope have been to OPC’s disadvantage in competing with DCFs as dispersion compensators. However, it is noted that the performance of optical phase conjugators has recently been and will continue to be improved significantly [13, 14]. Moreover, we argue that OPC and modern DCFs may work together nicely to complement each other’s functionalities. On one hand, transmission fibers and DCFs may be combined into fiber spans with zero dispersion slope, then OPC is able to equalize the residual dispersion and the slope of dispersion slope among such spans. On the other hand, the flexible designs and various choices in the dispersion parameters of specialty fibers, in particular DCFs, make it possible to construct fiber transmission lines that manifest “scaled symmetries” about the OPC, which are desired properties to effectively suppress fiber nonlinearities [15, 16, 17].

Based on the nonlinear Schrödinger equation (NLSE), it has been shown that OPC enables one fiber transmission line to propagate inversely (thus to restore) an optical signal that is nonlinearly distorted by the other, when the two fiber lines are mirror-symmetric about the OPC in the scaled sense [11, 15, 17]. Preliminary experiments have confirmed such effect of nonlinear compensation [11, 12]. Unfortunately, the mirror symmetry requires that the conjugating fiber segments have opposite loss/gain coefficients, the same sign for the second-order dispersions, and opposite third-order dispersions. These conditions are not conveniently fulfilled in many practical fiber transmission systems. In particular, a mirror-symmetric signal power profile is possible only when some transmission fibers are made distributively amplifying by means of distributed Raman pumping [18] or using distributed Er-doped fiber amplifiers (EDFAs) [19], so to obtain a constant net gain in correspondence to the loss coefficient of other fibers, or all fibers are rendered lossless. Recent experiments [20, 21, 22] have indeed demonstrated near constant-power or low power-excursion optical transmissions. However, there are still concerns of cost, reliability, and double-Rayleigh-scattering noise with distributive Raman amplification [18]. For any distributive amplifier, the loss of pump power makes it difficult to maintain a constant gain in a long transmission fiber. Consequently, the mismatch in signal power profiles degrades the result of nonlinear compensation. Yet another shortcoming of the previous schemes [11, 12] is that they do not compensate higher-order dispersions, which could turn into a significant limitation in wide-band transmission systems. By contrast, a recently proposed method

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of nonlinearity compensation using scaled translational symmetry requires that the conjugating fiber segments have the same sign for the loss/gain coefficients, opposite second-order dispersions, and the same sign for the third-order dispersions [16, 17]. Such conditions are naturally satisfied, at least approximately, in conventional fiber transmission systems, where, for example, a standard single-mode fiber (SMF) may be paired with a DCF as conjugating counterparts. In Refs. [16, 17], we have briefly touched upon the basic idea and feasibility of nonlinearity compensation using scaled translational symmetry. In this paper, we shall present an extensive and systematic study of the theory and practical applications of scaled translational symmetry in fiber transmission systems for nonlinearity compensation. Most importantly, we demonstrate that the combination of scaled nonlinearity, translational symmetry, OPC, and slope-matching dispersion compensation makes our proposals of nonlinearity compensation rather practical and highly performing. The notion of scaling fiber nonlinearity is not entirely new. The concept was proposed and utilized by Watanabe *et al*. in their 1996 paper [11], which however was limited to the mirror-symmetric configuration, and presented embodiments using segmented fibers which might not be convenient to implement in practice. Even though we may be the first to emphasize the concept and importance of scaled translational symmetry to nonlinearity compensation in fiber transmission lines [16, 17], it was noted previously by Marhic *et al*. [23] that two fibers having opposite dispersions and with OPC in the middle may compensate each other's Kerr nonlinear effects. However, Ref. [23] did not discuss any practical embodiment, nor did it mention the scaling of nonlinearity which is indispensable for practical implementations of translationally symmetric transmission lines. Both Refs. [11] and [23] had the effect of dispersion-slope neglected, and did not worry about the Raman effect among wavelength-division multiplexed (WDM) channels. By contrast, this present paper strives for the most generality, and it might be one of the early proposals for optimizing fiber transmission systems by combining the necessary and available four elements, namely, scaled nonlinearity, translational symmetry, OPC, and slope-matching dispersion compensation. It is this combination that signifies the present work and makes our proposals of nonlinearity compensation rather practical and highly performing.

2. Principles of dispersion and nonlinearity compensation using OPC

Dispersion equalization by OPC may be explained nicely using transfer functions in the frequency domain [24]. Optical signals in a fiber, possibly of many channels wavelength-division multiplexed together, may be described by a total electrical field $E(t) = A(t)\exp(-i\omega_0 t)$, with ω_0 being the center frequency of the optical band, and $A(t)$ being the slow-varying envelope [17, 25]. Equivalently, the total optical signal may be represented by the Fourier transform of the envelope,

$$\tilde{A}(\omega) \stackrel{\text{def}}{=} \mathscr{F}A(t) = \int E(t)\exp[i(\omega_0+\omega)t]dt, \tag{1}$$

where $\mathscr{F}$ denotes the operation of Fourier transform. Leaving aside the loss/gain and neglecting the nonlinearities, the linear dispersive effect of a fiber transmission line is described by a multiplicative transfer function,

$$H(\omega) = \exp\left(i\sum_{k=2}^{+\infty}\frac{b_k\omega^k}{k!}\right), \tag{2}$$

with

$$b_k = \int \beta_k(z)dz, \ \ \forall\, k \geq 2, \tag{3}$$

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being the dispersions accumulated along the fiber length,

$$\beta_k(z) \stackrel{\text{def}}{=} \left.\frac{\partial^k \beta(z,\omega)}{\partial \omega^k}\right|_{\omega=\omega_0}, \ \forall\, k \geq 2, \tag{4}$$

being the z-dependent dispersion coefficients of various orders [1, 17, 25], and $\beta(z,\omega)$ being the ω-dependent propagation constant of optical wave in the fiber. Because of the definition in terms of derivatives, β_2 may be called the second-order dispersion (often simply dispersion in short), while β_3 may be called the third-order dispersion, so on and so forth. The engineering community has used the term dispersion for the parameter $D = dv_g^{-1}/d\lambda$, namely, the derivative of the inverse of group-velocity with respect to the optical wavelength, and dispersion slope for $S = dD/d\lambda$ [1]. Although β_2 and D are directly proportional to each other, the relationship between β_3 and S is more complicated. To avoid confusion, this paper adopts the convention that dispersion and second-order dispersion are synonyms for the β_2 parameter, while dispersion slope and third-order dispersion refer to the same β_3 parameter, and similarly the slope of dispersion slope is the same thing as the fourth-order dispersion β_4.

A fiber line with dispersion parameters in Eq. (4) transforms a signal $\tilde{A}(\omega)$ into $H(\omega)\tilde{A}(\omega)$, while OPC acts as an operator that changes the same signal into $\mathsf{OPC}[\tilde{A}(\omega)] = \tilde{A}^*(-\omega)$. Consider two fiber transmission lines that are not necessarily identical, but nevertheless have accumulated dispersions satisfying the conditions,

$$b_k^R = (-1)^k b_k^L, \ \forall\, k \geq 2, \tag{5}$$

so that $H_R(\omega) = H_L(-\omega)$, where the super- and sub-scripts L, R are used to distinguish the two fiber lines. When OPC is performed in the middle of the two fiber lines, the entire setup transforms an input signal $\tilde{A}(\omega)$ into,

$$H_R(\omega)\mathsf{OPC}[H_L(\omega)\tilde{A}(\omega)] = H_R(\omega)H_L^*(-\omega)\tilde{A}^*(-\omega) = \tilde{A}^*(-\omega). \tag{6}$$

If $\tilde{A}(\omega)$ is the Fourier transform of $A(t)$, then the output signal $\tilde{A}^*(-\omega)$ corresponds to $A^*(t)$ in the time domain, which is an undistorted replica of the input signal $A(t)$ up to complex conjugation. This proves that the dispersion of a transmission line with OPC in the middle may be compensated over a wide bandwidth, when the dispersion coefficients of the odd orders on the two sides of OPC, b_{2k+1}^L and b_{2k+1}^R with $k \geq 1$, in particular the third-order dispersions b_3^L and b_3^R, are both compensated to zero, or they are exactly opposite to each other, while the even-order dispersion coefficients are the same on both sides. If a link has $b_3^R = -b_3^L$, or even $b_3^R = b_3^L = 0$, then it is compensated at least up to and including the fourth-order dispersion b_4. It is worth pointing out that the center frequency of the signal band may be shifted by the OPC from ω_0^L on the left side to ω_0^R on the right side, $\omega_0^L \neq \omega_0^R$, and the dispersion parameters on the two sides of OPC are defined with respect to the corresponding center frequencies.

To compensate the nonlinearity of transmission fibers, our method of using scaled translational symmetry [16, 17] requires that the conjugating fiber segments have the same sign for the loss/gain coefficients, opposite second-order dispersions, and the same sign for the third-order dispersions. Such conditions are naturally satisfied, at least approximately, in conventional fiber transmission systems, where, for example, an SMF may be paired with a DCF as conjugating counterparts. The symmetry is in the scaled sense, because the lengths of the fibers and the corresponding fiber parameters, including the fiber loss coefficients and dispersions, as well as the Kerr and Raman nonlinear coefficients, are all in proportion, and the proportional ratio may not be 1. The symmetry is called translational, because the curves of signal power variation along the fiber keep the similar shape, albeit scaled, when translated from the left to the right side of OPC, as depicted in Fig. 1, so do the curves of any above-mentioned fiber parameter if plotted against the fiber length. The fundamental discovery is that two fiber lines

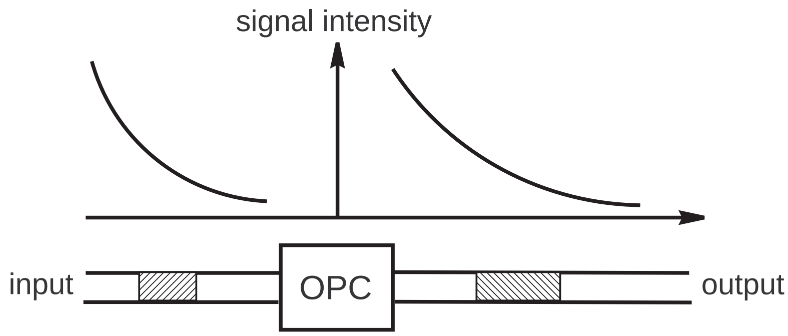

Fig. 1. Two fiber spans in translational symmetry about an optical phase conjugator. The shaded areas represent two typical fiber segments that are in scaled translational symmetry about the conjugator.

translationally symmetric about the OPC are able to cancel each other's nonlinearities up to the first-order perturbation. To understand the principle, imagine two fiber lines with opposite nonlinear coefficients but identical linear parameters of dispersion and loss/gain. It turns out that the nonlinear effects of the two are compensated up to the first-order perturbation when they are used in cascade. For an optical signal of the form,

$$E(z,t) = A(z,t)\exp\left[i\int^{z}\beta_0(\zeta)d\zeta - i\omega_0 t\right], \tag{7}$$

which may be of a single time-division multiplexed channel or a superposition of multiple WDM channels, the propagation in an optical fiber of length L is governed by the NLSE [17, 25],

$$\begin{aligned}\frac{\partial A(z,t)}{\partial z} + \sum_{k=2}^{+\infty}\frac{i^{k-1}\beta_k(z)}{k!}\left(\frac{\partial}{\partial t}\right)^k A(z,t) + \frac{\alpha(z)}{2}A(z,t) = \\ i\gamma(z)|A(z,t)|^2 A(z,t) + i\left[g(z,t)\otimes|A(z,t)|^2\right]A(z,t),\end{aligned} \tag{8}$$

$\forall\, z \in [0,L]$, in the retarded reference frame where the origin $z=0$ moves along the fiber at the signal group-velocity. $\alpha(z)$ is the loss/gain coefficient of the fiber, $\{\beta_k(z)\}_{k\geq 2}$ are its dispersion parameters [26], $\gamma(z)$ is the Kerr nonlinear coefficient of the fiber, $g(z,t)$ is the impulse-response of the Raman gain, and $\otimes$ denotes functional convolution [17, 25]. Note that the fiber parameters are allowed to be z-dependent, that is, they may vary along the length of the fiber. Had there been no nonlinearity, $\gamma(z) = g(z,\cdot) \equiv 0$, Eq. (8) would reduce to,

$$\frac{\partial A(z,t)}{\partial z} + \sum_{k=2}^{+\infty}\frac{i^{k-1}\beta_k(z)}{k!}\left(\frac{\partial}{\partial t}\right)^k A(z,t) + \frac{\alpha(z)}{2}A(z,t) = 0, \tag{9}$$

which could be solved analytically using, for example, the method of Fourier transform. A signal $\tilde{A}(z_1,\omega)$ at $z = z_1$ would be transformed into $\tilde{A}(z_2,\omega) = H(z_1,z_2,\omega)\tilde{A}(z_1,\omega)$ at $z_2 \geq z_1$, where $H(z_1,z_2,\omega)$ is defined as,

$$H(z_1,z_2,\omega) \stackrel{\text{def}}{=} \exp\left[i\sum_{k=2}^{+\infty}\frac{\omega^k}{k!}\int_{z_1}^{z_2}\beta_k(z)dz - \frac{1}{2}\int_{z_1}^{z_2}\alpha(z)dz\right]. \tag{10}$$

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In the time domain, the signals are related linearly as $A(z_2,t) = \mathsf{P}(z_1,z_2)A(z_1,t)$, with the linear operator $\mathsf{P}(z_1,z_2)$ given by,

$$\mathsf{P}(z_1,z_2) \stackrel{\text{def}}{=} \mathscr{F}^{-1}H(z_1,z_2,\omega)\mathscr{F}, \tag{11}$$

namely, $\mathsf{P}(z_1,z_2)$ is the concatenation of three linear operations: Firstly Fourier transform is applied to convert a temporal signal into a frequency signal, which is then multiplied by the transfer function $H(z_1,z_2,\omega)$, finally the resulted signal is inverse Fourier transformed back into the time domain. In terms of the impulse response,

$$h(z_1,z_2,t) \stackrel{\text{def}}{=} \mathscr{F}^{-1}[H(z_1,z_2,\omega)], \tag{12}$$

$\mathsf{P}(z_1,z_2)$ may also be represented as,

$$\mathsf{P}(z_1,z_2) = h(z_1,z_2,t)\otimes, \tag{13}$$

namely, the action of $\mathsf{P}(z_1,z_2)$ on a time-dependent function is to convolve the function with the impulse response. All linear operators $\mathsf{P}(z_1,z_2)$ with $z_1 \leq z_2$, also known as propagators, form a semigroup [27] for the linear evolution Eq. (9). However, the existence of nonlinear terms in Eq. (8) makes the equation much more difficult to solve. Fortunately, when the signal power is not very high so that the nonlinearity is weak and may be treated as perturbation, the output from a nonlinear fiber line may be represented by a linearly dispersed version of the input, plus nonlinear distortions expanded in power series of the nonlinear coefficients [28].

In practical transmission lines, although the end-to-end response of a long link may be highly nonlinear due to the accumulation of nonlinearity through many fiber spans, the nonlinear perturbation terms of higher orders than the first are usually negligibly small within each fiber span. Up to the first-order perturbation, the signal $A(z_2,t)$ as a result of nonlinear propagation of a signal $A(z_1,t)$ from z_1 to $z_2 \geq z_1$, may be approximated using,

$$A_0(z_2,t) = \mathsf{P}(z_1,z_2)A(z_1,t), \tag{14}$$

$$\begin{aligned} A_1(z_2,t) = & \int_{z_1}^{z_2} \mathsf{P}(z,z_2)\left\{i\gamma(z)|A_0(z,t)|^2 A_0(z,t)\right. \\ & \left. + i\left[g(z,t)\otimes|A_0(z,t)|^2\right] A_0(z,t)\right\} dz, \end{aligned} \tag{15}$$

where $A(z_2,t) \approx A_0(z_2,t)$ amounts to the zeroth-order approximation which neglects the fiber nonlinearity completely, whereas the result of first-order approximation $A(z_2,t) \approx A_0(z_2,t) + A_1(z_2,t)$ accounts in addition for the lowest-order nonlinear products integrated over the fiber length. The term $A_1(\cdot,t)$ is called the first-order perturbation because it is linearly proportional to the nonlinear coefficients $\gamma(\cdot)$ and $g(\cdot,t)$. Now back to the consideration of two fiber lines with opposite nonlinear coefficients but identical linear parameters of dispersion and loss/gain. It is obvious from Eqs. (10-15) that the two fiber lines induce opposite first-order nonlinear distortions to otherwise the same linear signal propagation (zeroth-order approximation). If the overall dispersion of each line is compensated to zero and the signal loss is made up by linear optical amplifiers, then the two lines may be used in cascade, and the same perturbation argument may be applied to the resulting two-span transmission line to show that the fiber nonlinearity is annihilated up to the first-order perturbation. More generally, one fiber may have its linear parameters scaled by a common factor and its nonlinear coefficients scaled by another factor, then the length of the fiber may be scaled inversely proportional to the linear parameters, and the signal power may be adjusted accordingly to yield the same strength of nonlinear interactions. This is coined scaled nonlinearity [15, 16, 17], which enables nonlinear

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compensation among fibers of different types. The problem is that an optical fiber with negative nonlinear coefficients may be only fictitious. It does not exist naturally.

For a fictitious fiber of length L and with parameters as those in Eq. (8), the Kerr nonlinear coefficient $\gamma(\cdot)$ is negative-valued, and the Raman gain is reversed in the sense that it induces optical power flow from lower to higher frequencies, which obviously will not happen normally. Fortunately, such fictitious fiber may be simulated by an ordinary fiber with the help of OPC. An ordinary fiber of length L/R may be found with parameters α', $\{\beta'_k\}_{k\geq 2}$, γ', g' satisfying the following rules of scaling,

$$\begin{aligned}
\alpha'(z) &= R\alpha(Rz), && (16)\\
\beta'_k(z) &= (-1)^{k-1}R\beta_k(Rz),\ \ \forall\, k\geq 2, && (17)\\
\gamma'(z) &= -Q\gamma(Rz), && (18)\\
g'(z,t) &= -Qg(Rz,t),\ \ \forall\, t\in(-\infty,+\infty), && (19)
\end{aligned}$$

$\forall\, z\in[0,L/R]$, where $R>0$, $Q>0$ are scaling factors. In this ordinary fiber, the NLSE of signal propagation is,

$$\begin{aligned}
&\frac{\partial A'(z,t)}{\partial z}+\sum_{k=2}^{+\infty}\frac{i^{k-1}\beta'_k(z)}{k!}\left(\frac{\partial}{\partial t}\right)^k A'(z,t)+\frac{\alpha'(z)}{2}A'(z,t)=\\
&\quad i\gamma'(z)|A'(z,t)|^2A'(z,t)+i\left[g'(z,t)\otimes|A'(z,t)|^2\right]A'(z,t), \qquad (20)
\end{aligned}$$

$\forall\, z\in[0,L/R]$, that is,

$$\begin{aligned}
&\frac{\partial A'(z,t)}{R\partial z}+\sum_{k=2}^{+\infty}\frac{(-i)^{k-1}\beta_k(Rz)}{k!}\left(\frac{\partial}{\partial t}\right)^k A'(z,t)+\frac{\alpha(Rz)}{2}A'(z,t)=\\
&-iQR^{-1}\gamma(Rz)|A'(z,t)|^2A'(z,t)-iQR^{-1}\left[g(Rz,t)\otimes|A'(z,t)|^2\right]A'(z,t), \qquad (21)
\end{aligned}$$

$\forall\, z\in[0,L/R]$. After a substitution,

$$A'(z,t)=\sqrt{R/Q}A^*(Rz,t), \tag{22}$$

then a change of variable $Rz\rightarrow z$, and finally taking the complex conjugate of the whole equation, Eq. (21) becomes mathematically identical to Eq. (8). Equation (22) is actually the scaling rule for the signal amplitudes. The physical implication is that, if a signal $A(0,t)$ is injected into the fictitious fiber and the complex conjugate signal $\sqrt{R/Q}A^*(0,t)$ is fed to the ordinary fiber, then the signal at any point $z\in[0,L/R]$ in the ordinary fiber is $\sqrt{R/Q}A^*(Rz,t)$, which is the complex conjugate of the signal at the scaled position Rz in the fictitious fiber. In particular, the output signals are $A(L,t)$ and $\sqrt{R/Q}A^*(L,t)$ from the fictitious and the ordinary fibers respectively. With signal power scaled by the factor R/Q, the ordinary fiber with two phase conjugators installed at its two ends performs exactly the same nonlinear signal transformation as the fictitious fiber. In practice, the phase conjugator at the output end of the ordinary fiber may be omitted, as most applications would not differentiate between a signal and its complex conjugate.

The analysis has convinced us that OPC may be used to compensate fiber nonlinearities between two transmission lines that are in scaled translational symmetry. Both the Kerr and Raman nonlinearities may be suppressed simultaneously if a proportional relation is maintained between the γ and g parameters as in the scaling rules Eq. (18) and Eq. (19). When Eqs. (18) and (19) can not be fulfilled simultaneously, either the Kerr or the Raman nonlinearity may be primarily targeted for compensation depending upon the actual application. For a translational

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symmetry between two fibers with opposite dispersions, the scaling rule Eq. (16) requires the same sign for the loss/gain coefficients of the two fibers, which is a convenient condition to meet by the natural fiber losses. This is in contrast to the mirror symmetry between two fiber segments that requires an amplifying segment correspond to a lossy one and vice versa. Fibers may be designed and fabricated with the requirements of scaled symmetry taken into consideration. For a given piece of fiber, the loss coefficient may be intentionally increased to meet the scaling rule. The extra loss may be induced by, for example, macro-bending [29] the fiber or writing long-period Bragg gratings into the fiber. Macro-bending may be built in a lumped fiber module having the fiber coiled tightly with a suitable radius. Also discrete fiber coils or Bragg gratings may be implemented periodically along the length of a fiber to approximate a continuous uniform loss coefficient. More sophisticatedly, Raman pumps may be employed to induce gain or loss to the optical signals depending upon the pump frequencies being higher or lower than the signal band, so to alter the effective gain/loss coefficient of the fiber. Even though it is rather difficult to change the dispersion of a given fiber, OPC is capable of shifting the center frequency of the signal band, which can fine-tune the effective dispersion at the center of the signal band, so long as the fiber has a non-zero dispersion slope. Even though most fibers are made of similar materials with similar nonlinear susceptibilities, their guided-wave nonlinear coefficients measured in $\mathrm{W}^{-1}\mathrm{km}^{-1}$ could be quite different due to the wide variation of modal sizes. As a consequence, the signal powers in two conjugate fibers may differ by several dB as required by the scaling rule Eq. (22) for translational symmetry. Alternatively, by taking advantage of the additivity of first-order nonlinear perturbations, it is possible to adjust the signal powers in different fiber spans only slightly, such that one span of a highly-nonlinear type may compensate several fiber spans of another type with weaker nonlinearity. This method may be called “one-for-many” (in terms of fiber spans) nonlinearity compensation.

It should be noted that the suitability of compensating nonlinearities among lossy fibers does not exclude the method of translational symmetry from applying to systems with amplifying fibers due to Raman pumping [18, 20, 21, 22, 30] or rare-earth-element doping [19]. The translational method applies to these systems equally well, provided that an amplifying fiber is brought into translational symmetry with respect to another fiber with gain. In fact, if two fibers with their intrinsic loss coefficients satisfying the scaling rule Eq. (16), then the power of the Raman pumps (forward or backward) to them may be adjusted properly to yield effective gain/loss coefficients satisfying the same rule of Eq. (16). In particular, Raman pumped DCFs [31, 32] may be conveniently tuned translationally symmetric to a Raman pumped transmission fiber. For systems suffering considerable nonlinear penalties originated from long EDFAs [33], the penalties may be largely suppressed by arranging the amplifiers into conjugate pairs with scaled translational symmetry about the OPC. The nonlinear and gain coefficients as well as the signal amplitudes in the amplifying fibers should obey the scaling rules. If the dispersions of the amplifying fibers are not negligible, they should be designed to satisfy the scaling rules as well. Finally, it is also necessary to note the limitation of nonlinearity compensation using scaled translational symmetry. That is, the method can only compensate the first-order nonlinear interactions among the optical signals. The higher-order nonlinear products are not compensated, nor is the nonlinear mixing between transmitted signals and amplifier noise. The accumulation of uncompensated higher-order nonlinearities and nonlinear signal-noise mixing would eventually upper-bound the amount of signal power permitted in the transmission fibers, so to limit the obtainable signal-to-noise ratio, and ultimately limit the product of data capacity and transmission distance.

3. Optimal setups of fiber-optic transmission lines

Having established the basic principles of dispersion equalization and nonlinearity compensation using OPC and scaled translational symmetry, we shall now discuss practical designs of fiber systems for long-distance transmissions, with realistic (commercially available) DCFs and transmission fibers that are optimally configured according to the basic principles of simultaneous compensation of dispersion and nonlinearity. A long-distance transmission line may consist of many fiber spans, each of which may have transmission and dispersion-compensating fibers. Two fibers with opposite (second-order) dispersions may be tuned translationally symmetric to each other about a phase conjugator. For optimal nonlinearity compensation, the fiber parameters and the signal amplitudes should be adjusted to meet the conditions of translational symmetry, often approximately, not exactly, because of the dispersion slopes [17]. In particular, if one fiber span has a positive-dispersion (+D) fiber followed by a negative-dispersion (-D) fiber, then the counterpart span has to place the -D fiber before the +D fiber, in order to achieve an approximate translational symmetry between the two fiber spans. When two fiber spans are translationally symmetric about an optical phase conjugator, one span is called the translational conjugate to the other about the OPC. As argued above, OPC is able to equalize dispersion terms of even orders. So the two parts of a transmission line with OPC in the middle should have the same amount of b_2 and b_4 but exactly opposite b_3, or both have $b_3 = 0$, where the b-parameters are defined in Eq. (3). In a more restrictive implementation, each fiber span consists of +D and -D fibers with the total dispersion slope compensated to zero. The +D and -D fibers in each span need not to match their dispersions and slopes simultaneously. It is sufficient to fully compensate b_3, while leaving residual even-order terms b_2 and b_4. Two conjugate spans would be configured as +D followed by -D fibers and -D followed by +D fibers respectively. The two conjugate spans may not be exactly the same in length, and they may have different integrated dispersion terms of the even orders. The two types of fiber spans may be mixed and alternated on each side of the OPC, so that the two sides have the same total b_2 and b_4. Transmission lines with such dispersion map are convenient to plan and manage. However, it is worth noting that the present method of simultaneous compensation of dispersion and nonlinearity applies to other dispersion maps as well, where the period of dispersion compensation may be either shorter [34] or longer [35] than the amplifier spacing, or the fiber spans may vary widely in length and configuration. Regardless of the dispersion map, wide-band dispersion compensation could be achieved in a transmission line with middle-span OPC so long as the dispersion terms of the two sides of OPC satisfy Eq. (5), and pairs of conjugate fiber spans could have their nonlinearities cancelled up to the first-order perturbation as long as the scaling rules Eqs. (16-19) and Eq. (22) are well observed.

As a result of power loss, the nonlinear response of a long piece of fiber becomes insensitive to the actual fiber length so long as it far exceeds the effective length [2] defined as $L_{\text{eff}} = 1/\alpha$, where α is the loss coefficient. So fiber spans consisting of the same types of fibers but with different lengths could contribute the same amount of nonlinearity if the input powers are the same. That all fiber spans contribute the same nonlinearity makes it possible for various spans with different lengths to compensate each other's nonlinear effects. It is straightforward to extend the same argument to fiber spans with scaled parameters and signal powers. The conclusion is that scaled fiber spans could induce approximately the same amount of nonlinear distortion to optical signals, which is insensitive to the varying span lengths, provided that the length of each fiber span is much longer than its own effective length defined by the inverse of the loss coefficient. The main advantage is that the fiber spans may be arbitrarily paired for nonlinearity compensation regardless of their actual lengths. This is good news to terrestrial and festoon systems, where the span-distance between repeaters may vary according to the geographical conditions. When the dispersion of each fiber span is not fully compensated, it is desirable to

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fine-tune (slightly elongate or shorten) the lengths of transmission fibers or DCFs such that all spans have the same amount of residual dispersion. As a consequence, fiber spans of different lengths and possibly consisting of different types of fibers become truly equivalent in two all-important aspects of signal propagation: nonlinearity and accumulated dispersion. Certainly, if the above-mentioned method of "one-for-many" nonlinearity compensation is employed, the residual dispersion of the highly nonlinear span should also be multiplied by the same integer factor. Last but not least, when scaling fiber parameters and signal amplitudes to have two fiber spans inducing the same or compensating nonlinear effects, it is only necessary to make sure that the scaling rules Eqs. (16-19) and Eq. (22) are fulfilled in portions of transmission fibers experiencing high levels of signal power. Elsewhere, the scaling rules may be loosened or neglected when the signal power is low.

Despite the translational symmetry between the constituent fibers of two conjugate spans, it is advantageous to order many conjugate spans in a mirror-symmetric manner about the OPC, especially when all the spans are not identical. The local nonlinearity within each span is usually weak such that the nonlinear perturbations of higher orders than the first may be neglected, even though a strong nonlinearity may be accumulated through many fiber spans. Within the applicability of first-order perturbation for approximating the nonlinearity of each fiber span, it may be argued using mathematical induction that the nonlinearity of multiple spans in cascade is also compensated up to the first-order perturbation, because of the mirror-symmetric arrangement of fiber spans about the OPC. The spans may be labelled from left to right by $-N, \cdots, -2, -1, 1, 2, \cdots, N$, with OPC located between span -1 and span 1. And one may denote by z_0 and z_0' the beginning and end positions of the section of OPC, while labelling the beginning and end points of span n by z_n and z_n', where $z_n' = z_{n+1}$, $\forall\, n \in [-N, N-1]$. There may be three variations for a mirror-symmetric configuration of pairs of fiber spans in scaled translational symmetry, depending upon whether the dispersion in each span is compensated to zero, and if not, how the dispersion is managed. In the first case, all spans are compensated to zero dispersion, as shown in Fig. 2 for the case of $N = 3$. It is required that, $\forall\, n \in [1, N]$, spans $-n$ and n should be conjugate, that is translationally symmetric, to each other. The first-order nonlinear perturbations of spans 1 and -1 cancel each other due to the translational symmetry and the OPC, so the optical path from z_{-1} to z_1' is equivalent to an ideal linear transmission line with OPC in the middle, if higher-order nonlinear perturbations are neglected. It follows that the signal input to span 2 at z_2 is approximately the complex conjugate of that input to span -2 at z_{-2}, apart from the nonlinear perturbation due to span -2. So the translational symmetry between spans 2 and -2 about the OPC annihilates their nonlinearities up to the first-order perturbation. Using mathematical induction, assuming that the optical path from z_{-n} to z_n', $1 < n < N$, is equivalent to an ideal linear transmission line with OPC in the middle, then spans $n+1$ and $-n-1$ see input signals at z_{n+1} and z_{-n-1} that are approximately complex conjugate to each other, so their first-order nonlinear effects cancel each other out due to the translational symmetry and OPC. The optical path from z_{-n-1} to z_{n+1}' is linearized and equivalent to an ideal linear transmission line with OPC in the middle. This inductive argument applies as long as the accumulation of nonlinear perturbations of higher-orders than the first is still negligible and the nonlinear mixing of amplifier noise into signal hasn't grown significantly.

In the second case, the fiber spans may have non-zero residual dispersion, as shown in Fig. 3 for the case of $N = 3$. It is required that, $\forall\, n \in [1, N]$, spans $-n$ and n should be in a translational symmetry approximately, while the residual dispersion of span $n-1$ should be approximately the same as span $-n$, $\forall\, n \in [2, N]$. Pre- and post-dispersion compensators are employed to equalize the residual dispersion. The pre-dispersion may set the total dispersion to zero immediately before OPC, and a dispersion conditioner at the site of OPC ensures that the signal input to span 1 is approximately the complex conjugate of that input to span -1, apart from the non-

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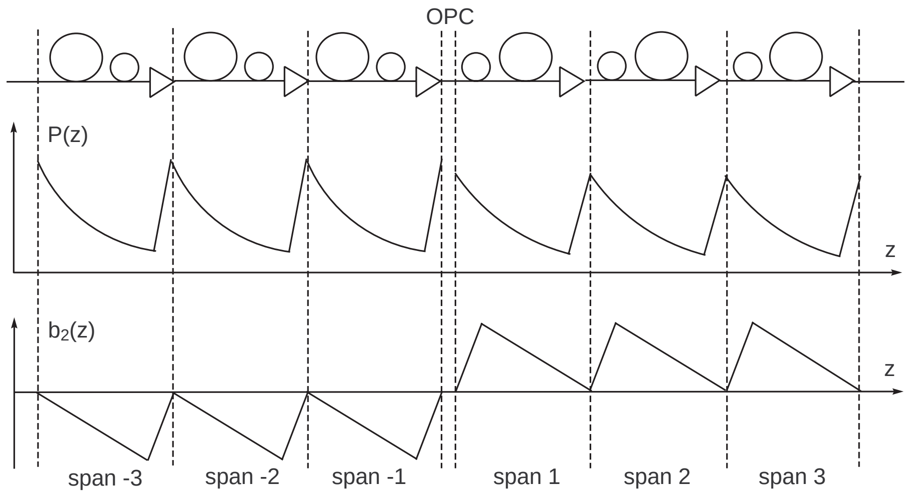

Fig. 2. A mirror-symmetric configuration of pairs of fiber spans in scaled translational symmetry, with the dispersion in each span compensated to zero. Top: schematic arrangement of fibers and amplifiers with respect to OPC. Middle: map of signal power $P(z)$ along the propagation distance z. Bottom: map of accumulated dispersion $b_2(z)$ along the propagation distance z.

linear perturbation due to span -1. Figure 3 shows a dispersion conditioner placed immediately after OPC, with the amount of dispersion equal to the residual dispersion in span -1. The three thicker line segments in the dispersion map represent the effects of the pre- and post-dispersion compensators as well as the dispersion conditioner. So the transmission line has been designed such that the accumulated dispersions from z_{-n} to z_n, $n \in [1,N]$, are fully compensated by virtue of OPC, and for each $n \in [1,N]$, the fiber span from z_{-n} to z'_{-n} is translationally symmetric to the fiber span from z_n to z'_n, namely, the parameters of the two fiber spans satisfy the scaling rules of Eqs. (16-19), at least approximately. Leaving aside the fiber nonlinearity, such dispersion map ensures that the optical signals at z_{-n} and z_n are complex conjugate to each other, then the signal amplitudes may be properly scaled such that Eq. (22) is also satisfied. As a result, all conditions are fulfilled for the fiber spans from z_{-n} to z'_{-n} and from z_n to z'_n to compensate their fiber nonlinearities up to the first-order perturbation, for each $n \in [1,N]$. The first-order nonlinear perturbations of spans 1 and -1 cancel each other due to the translational symmetry and OPC, so the optical path from z_{-1} to z'_1 is equivalent to an ideal linear transmission line with OPC in the middle and some accumulated dispersion at z'_1 due to span 1. Since this amount of dispersion is equal to that of span -2, the signal input to span 2 at z_2 is approximately the complex conjugate of that input to span -2 at z_{-2}, apart from the nonlinear perturbation due to span -2. So the translational symmetry between spans 2 and -2 about the OPC annihilates their nonlinearities up to the first-order perturbation. Using mathematical induction, assuming that the optical path from z_{-n} to z'_n, $1 < n < N$, is equivalent to an ideal linear transmission line with OPC in the middle and accumulated dispersion at the right end due to span n, which is the same amount of residual dispersion as of span $-n-1$, then spans $n+1$ and $-n-1$ see input signals at z_{n+1} and z_{-n-1} that are approximately complex conjugate to each other, so their first-order nonlinear effects cancel each other out due to the translational symmetry and OPC. The optical path from z_{-n-1} to z'_{n+1} is linearized and equivalent to an ideal linear transmission line with OPC in the middle and the dispersion of span $n+1$ at the right end. In the third case, the

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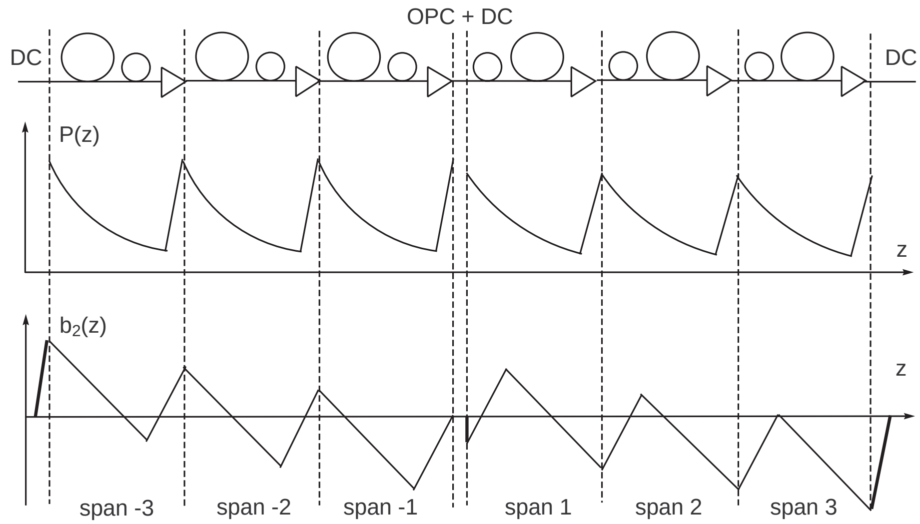

Fig. 3. A mirror-symmetric configuration of pairs of fiber spans in scaled translational symmetry, with non-zero residual dispersion in the spans. There are pre- and post-dispersion compensators (DCs), as well as a dispersion conditioner immediately after OPC. Top: schematic arrangement of fibers and amplifiers with respect to OPC. Middle: map of signal power $P(z)$ along the propagation distance z. Bottom: map of accumulated dispersion $b_2(z)$ along the propagation distance z.

fiber spans still have non-zero residual dispersion, but there is no dispersion conditioner placed immediately before or after OPC to compensate the residual dispersion of span -1. Instead, span 1 may play the role of the dispersion conditioner, and $\forall\, n \in [1,N]$, spans n and $-n$ need to have the same amount of residual dispersion, while spans n and $-n+1$, $\forall\, n \in [2,N]$, should be in a scaled translational symmetry approximately to have their nonlinearities compensated up to the first-order perturbation. This is in contrast to the requirement of the second case. The configuration is shown in Fig. 4 for the case of $N = 3$, where the two thicker line segments in the dispersion map represent the effects of the pre- and post-dispersion compensators. It may be shown using the same inductive argument that the transmission line is largely linearized, except that the nonlinear effects of spans 1 and $-N$, if any, are left uncompensated.

DCFs are widely used in modern fiber-optic transmission systems. A DCF may be coiled into a compact module at the amplifier site, or cabled as part of the transmission line. The performance of both types of DCFs has been greatly improved recently. There are now low-loss DCFs capable of (approximately) slope-matched dispersion compensation for various transmission fibers with different ratios of dispersion to dispersion-slope [3, 4], although there are always residual second-order and fourth-order dispersions after the slope is equalized [5, 6, 7]. For SMFs, namely standard single-mode fibers, the ratio of dispersion ($D \approx 16$ ps/nm/km @1550 nm) to dispersion slope ($S \approx 0.055$ ps/nm^2/km @1550 nm) is large, so that the relative change of dispersion is small across the signal band (≈ 40 nm in the C-band). The so-called reverse dispersion fibers (RDFs) are designed to compensate simultaneously the dispersion and dispersion slope of the SMFs. An RDF is not an ideal translational conjugate to an SMF, because their dispersion slopes do not obey the scaling rule in Eq. (17). However, their dispersions satisfy the corresponding scaling rule in Eq. (17) approximately, with only small deviations across the entire signal band (C or L). Therefore, a span having an SMF followed by an RDF on the left side

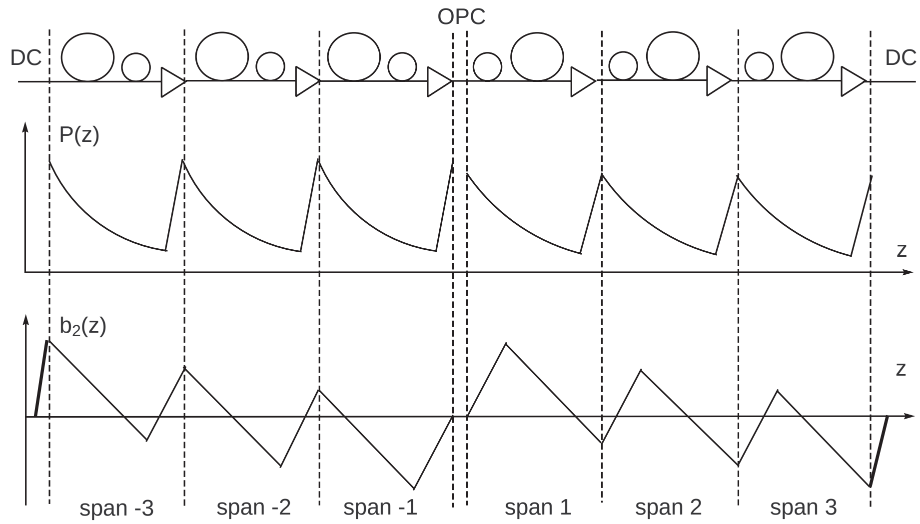

Fig. 4. A mirror-symmetric configuration of pairs of fiber spans in scaled translational symmetry, with non-zero residual dispersion in the spans. There are pre- and post-dispersion compensators (DCs) but no dispersion conditioner at the site of OPC. Top: schematic arrangement of fibers and amplifiers with respect to OPC. Middle: map of signal power $P(z)$ along the propagation distance z. Bottom: map of accumulated dispersion $b_2(z)$ along the propagation distance z.

of OPC may be brought into a translational symmetry, approximately, to a span having an RDF followed by an SMF on the right side of OPC, and vice versa. The indication is that OPC may be installed in the middle of conventional transmission lines with no or minimal modifications to achieve simultaneous wide-band dispersion compensation and nonlinearity suppression. The only requirements are that the signal power levels should be properly set in the fiber spans, and the SMFs/RDFs should be suitably arranged, to meet the scaling rules Eqs. (16-19) and Eq. (22) approximately for the translational symmetry between each pair of conjugate fiber spans, and to order the conjugate pairs of spans mirror-symmetrically about the OPC. It is noted that a recent paper [36] has independently proposed the combination of slope-matching DCF and OPC to suppress simultaneously the third-order dispersion and sideband instability due to fiber nonlinearity. However, the work [36] was limited to a single-channel system, considered only the suppression of sideband instability as an intra-channel nonlinear effect, and did not recognize the importance of scaling the nonlinearity (especially the signal power) in different fibers. By contrast, our method applies to wide-band WDM systems as well and is capable of suppressing both intra- and inter-channel nonlinear interactions, being them Kerr- or Raman-originated. Most importantly, we emphasize the importance of the scaling rules Eqs. (16-19) and Eq. (22) for optimal nonlinearity compensation.

Several non-zero dispersion-shifted fibers (NZDSFs) have also been developed for long-distance high-capacity transmissions. These fibers have reduced but non-zero dispersions across the operating band (C or L). Depending upon the sign of the dispersion (D in units of ps/nm/km), there are positive NZDSFs (+NZDSFs) and negative NZDSFs (-NZDSFs), but their dispersion-slopes are always positive. It becomes possible to bring a +NZDSF and a -NZDSF into a nearly perfect translational symmetry [37], because their oppositely signed dispersions and positively signed dispersion-slopes meet the exact requirements of the scaling rules in Eq.

(17). The dispersion-slope of the NZDSFs may be compensated by negative-slope DCFs. The DCFs do not have to (could not indeed) compensate the dispersion and dispersion-slope simultaneously for both the positive and negative NZDSFs. It is sufficient to equalize the accumulated dispersion-slope to zero on each side of the OPC, then the two sides may cancel their accumulated non-zero dispersions of the second and the fourth orders through OPC. To form a nonlinearity-compensating translational symmetry between a +NZDSF span and a -NZDSF span, the accumulated dispersion should be properly managed to ensure that the input signals to the +NZDSF and -NZDSF fibers are complex conjugate to each other, which is a necessary condition for nonlinearity cancellation. As long as these requirements are satisfied, there is really no limit as to how much residual (second-order) dispersion may be accumulated in each fiber span as well as on each side of the OPC. It may be difficult to find a fiber translationally symmetric to the slope-compensating DCF, because of its high negative dispersion-slope. However, we note that it is only necessary to have a scaled translational symmetry formed between portions of fibers carrying high signal power, elsewhere, such as in the slope-compensating DCFs, the scaling rules may be neglected when the signal power is low and the nonlinearity is insignificant. If the slope-compensating DCFs are cabled, they may be placed near the end of fiber spans where the signal power is low. Or if the DCFs are coiled into modules and co-located with the amplifiers, the signal power inside may be controlled at a low level to avoid nonlinearity. To minimize the noise-figure penalty in such DCF modules, the DCF may be distributively Raman pumped [18, 31, 32], or earth-element doped and distributively pumped [19], or divided into multiple segments and power-repeated by a multi-stage EDFA. The conclusion is that the method of OPC-based simultaneous compensation of dispersion and nonlinearity is perfectly suitable for transmission systems employing NZDSFs, and highly effective nonlinearity suppression may be expected in such systems due to the nearly perfect translational symmetry between the +NZDSFs and -NZDSFs. Finally, in the limit of vanishing (second-order) dispersion at the center of the signal band, the +NZDSF and -NZDSF converge to the same dispersion-shifted fiber (DSF), which is translationally symmetric to itself. Two identical DSF spans on the two sides of OPC are in perfect translational symmetry to cancel their nonlinearity up to the first-order perturbation. Again the dispersion-slope may be equalized by a DCF with negative dispersion-slope, and the residual second-order dispersion may be arbitrarily valued. Suppressing fiber nonlinearity happens to be highly desired in DSF-based transmission lines, as DSFs are arguably the most susceptible to nonlinear impairments [2].

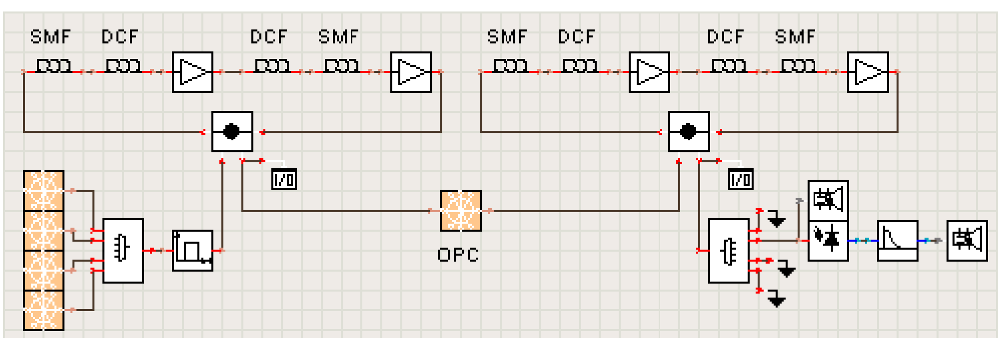

Fig. 5. A transmission line consisting of SMFs and slope-matching DCFs.

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4. Simulation results and discussions

To verify the proposed method of simultaneous compensation of dispersion and nonlinearity, we have carried out a series of numerical simulations using a commercial transmission simulator (VPItransmissionMaker$^{\mathrm{TM}}$, Virtual Photonics Inc.). Reference [17] has presented an example of SMFs and DCF modules with nearly perfect match of dispersion and slope. Here we consider a practical setup of SMFs and cabled DCFs with residual dispersion, as shown in Fig. 5. One type of span consists of 50 km SMF followed by 50 km DCF. The SMF has loss coefficient $\alpha = 0.2$ dB/km, effective mode area $A_{\mathrm{eff}} = 80\ \mu\mathrm{m}^2$, and dispersion parameters $\beta_2 = -20.47$ ps^2/km, $\beta_3 = 0.1238$ ps^3/km at 193.1 THz. The corresponding dispersion $D = 16$ ps/nm/km and slope $S = 0.055$ ps/nm^2/km. The DCF mimics a commercial RDF product [7], namely a reverse dispersion fiber, with parameters $(\alpha', A'_{\mathrm{eff}}, \beta'_2, \beta'_3) = (0.2, 30, 18, -0.1238)$, in the same units as for the SMF. The Kerr nonlinear index of silica $n_2 = 2.6 \times 10^{-20}$m^2/W. Practical DCFs often have a loss coefficient that is slightly higher than the SMFs, so the optimal design of the DCFs would have a dispersion $|D_{\mathrm{DCF}}|$ slightly higher than $|D_{\mathrm{SMF}}|$ proportionally according to the scaling rules Eq. (16) and Eq. (17). The conjugate span has 39.35 km DCF followed by SMF of the same length. Due to the smaller modal area, a lower power is injected into the DCF to generate the compensating nonlinearity, in accordance with the scaling rule for signal amplitudes in Eq. (22). The shortened span length is to balance the noise figure between the two types of spans. The two span types are also intermixed on each side of the OPC to balance the residual dispersions. Alternatively, all fiber spans may be the same in length, but the signal power injected to the DCF+SMF spans should be $3/8$ of that injected to the SMF+DCF spans, and the DCF+SMF spans would add more noise to the optical signal than the SMF+DCF spans. It is noted that the scaling rules are not obeyed at all in the second part of each span, that is, in the DCFs of SMF+DCF spans and in the SMFs of DCF+SMF spans. Fortunately, the second part of each span experiences low signal power, in which the nonlinear effect is negligible. Back to the setup of Fig. 5, where all EDFAs have the same noise figure of 5 dB, each fiber loop recirculates five times, that gives 1000 km worth of fiber transmission on each side of the OPC. The input are four 40 Gb/s WDM channels, return-to-zero (RZ) modulated with peak power 20 mW, channel spacing 200 GHz. Each RZ pulse generator consists of a continuous-wave laser followed by a zero-chirp modulator, which is over-driven to produce a pulse train with the amplitude proportional to $\cos\left(\frac{\pi}{2}\sin\pi\Omega t\right)$, where Ω is the bit rate. Therefore the duty cycle of the pulses is 33%, if defined as the ratio of pulse full-width-half-maximum to the time interval between adjacent bits. The optical multiplexer and demultiplexer consist of Bessel filters of the 7th order with 3dB bandwidth 80 GHz. The input data are simulated by pseudo random binary sequences of order 7, and the simulation time window covers 256 bits. The photo-detector is with responsivity 1.0 A/W and thermal noise 10.0 pA/$\sqrt{\mathrm{Hz}}$. The electrical filter is 3rd order Bessel with 3dB bandwidth 28 GHz. Figure 6 shows the received eye diagrams of the 2nd DEMUX channel. The top-right diagram shows the effect of nonlinearity compensation. For comparison, the result of a fictitious transmission where no fiber has any nonlinear effect is shown on the top-left of Fig. 6. To confirm that the suppression of nonlinearity is indeed due to the translational symmetry of conjugate spans about the OPC, the two diagrams at the bottom of Fig. 6 show simulation results of altered configurations: one setup has the same length of $50+50$ km for and the same input power level to both the SMF+DCF and the DCF+SMF spans, the other has on both sides of OPC identical 100-km SMF+DCF spans carrying the same signal power. Both altered setups suffer from severe nonlinear impairments.

For an example system using NZDSFs, we simulated a transmission line consisting of twenty 100-km fiber spans with OPC in the middle, as shown in Fig. 7, where each side of the OPC has a fiber loop circulated five times. In each circulation, the optical signals go through 100 km -NZDSF transmission followed by a two-stage EDFA with 10 km DCF in the middle, then 100

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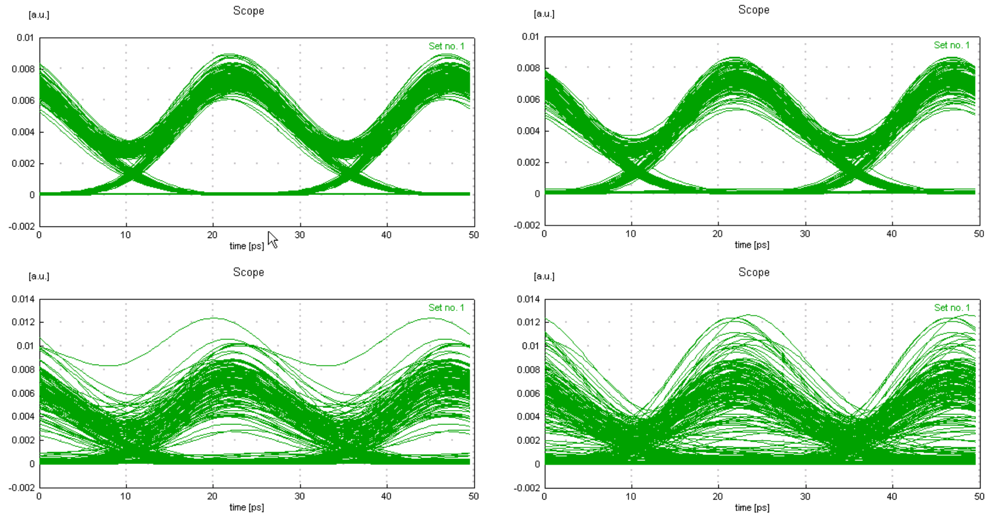

Fig. 6. Received eye diagrams of the 2nd DEMUX channel. Top row: transmission results of the setup in Fig. 5. Top-left: fiber nonlinearity is OFF, the signal is only impaired by amplifier noise. Top-right: fiber nonlinearity is ON, the signal distortion is only increased slightly. Bottom row: transmission results when the setup is modified, and the fiber nonlinearity is always ON. Bottom-left: fiber lengths of and input powers to the two types of spans are exactly the same. Bottom-right: all fiber spans are identical in length and input signal power as well as the ordering of fibers (SMF followed by DCF).

km +NZDSF transmission followed by the same two-stage EDFA and DCF. The +NZDSF has loss coefficient $\alpha = 0.2$ dB/km, dispersion $D = +4$ ps/nm/km and slope $S = 0.11$ ps/nm^2/km at 193.1 THz. The effective mode area is $A_{\text{eff}} = 70$ μm^2. The -NZDSF differs only by $D = -4$ ps/nm/km. The Kerr nonlinear index of silica $n_2 = 2.6 \times 10^{-20}$m^2/W. The two-stage EDFA has $11 + 15 = 26$ dB gain in total to repeat the signal power. The noise figure of each stage is 5 dB. The DCF has $\alpha = 0.6$ dB/km, $D = -40$ ps/nm/km, $S = -1.1$ ps/nm^2/km, $A_{\text{eff}} = 25$ μm^2, but nonlinearity neglected. The transmitting and receiving ends are the same as in the above SMF/DCF transmission. Input to the system are the same four-channel WDM signals, and the peak power of the 40 Gb/s RZ pules is also the same 20 mW. With their nonlinear effects neglected, the DCFs do not participate directly in nonlinearity compensation. Nevertheless, their compensation of the dispersion-slope of the NZDSFs enables the OPC to effectively compensate the dispersion over a wide frequency band, and helps to condition the optical signals such that the inputs to two conjugate NZDSFs are mutually complex conjugate. Note that the +NZDSF and -NZDSF spans are alternated on each side of the OPC to balance the accumulated dispersion between the two sides. Also note that the first -NZDSF span on the right side of OPC is designed to compensate the nonlinearity of the last +NZDSF span on the left side, and the second span on the right (+NZDSF) is to compensate the second last span (-NZDSF) on the left, so on and so forth. It is important for the +NZDSF spans to be well dispersion-compensated, so to ensure that the input signals to the two conjugate spans of a translationally symmetric pair are complex conjugate to each other, which is a necessary condition for nonlinearity cancellation. However, there is no limit as to how much residual dispersion may be in the -NZDSF spans. Alternatively, each fiber span may be a concatenation of + and - NZDSFs. One type of span may have a +NZDSF followed by a -NZDSF, then the conjugate span would

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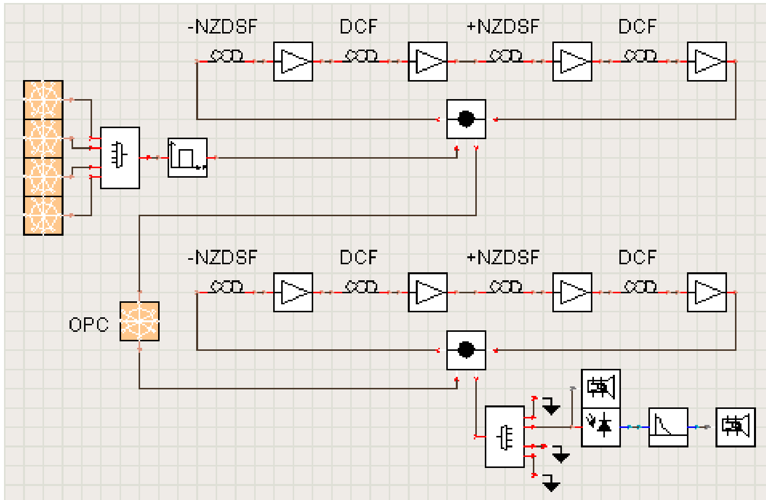

Fig. 7. A transmission line consisting of +NZDSFs, -NZDSFs, and DCFs compensating the dispersion slope.

consist of the same fibers reversely ordered. Consequently, all spans may use the same DCF for slope-compensating, and all accumulate the same dispersions of even orders. Figure 8 shows the received eye diagrams of the 2nd DEMUX channel. The top row shows the results of nonlinear transmission and the comparing fictitious transmission without fiber nonlinearity through the setup of Fig. 7. The effectiveness of nonlinear compensation is remarkable. By contrast, the bottom row of Fig. 8 shows severe degradations in the transmission performance, when all -NZDSFs are replaced by +NZDSFs, so that the transmission line consists of identical +NZDSF spans with DCFs compensating both the dispersion and the dispersion-slope. The highly effective nonlinearity compensation is expected as a result of the nearly perfect translational symmetry between the +NZDSF and -NZDSF spans. Furthermore, a nonlinearity-suppressed transmission line should manifest behaviors of a linear system to some extent. Typical linear behaviors include scalability and cascadability. Namely, using the same fiber spans and simply by raising the signal power, it is possible to further the transmission distance by increasing the number of fiber spans before/after the OPC (scaling up), or by cascading several OPC-compensated transmission lines all-optically (without optical to electrical and electrical to optical signal conversions in the middle). Both the scalability and the cascadability are confirmed via numerical simulations, as shown in Fig. 9, where one eye diagram is for a system with the number of spans doubled to 40 in total, and the other diagram is obtained when cascading two identical 20-span transmission lines of Fig. 7. The eye diagrams are still of the 2nd DEMUX channel.

To test the effectiveness of nonlinear compensation for DSFs, we evaluated numerically a transmission line consisting of twenty 50-km DSF spans with OPC in the middle, as shown in Fig. 10. Each span has 50 km DSF and at the end a two-stage EDFA with 5 km DCF in the middle. The DSF has loss $\alpha = 0.2$ dB/km, $D = 0$ ps/nm/km and $S = 0.08$ ps/nm^2/km at the center frequency 193.1 THz, $A_{\rm eff} = 50$ μm^2. The Kerr nonlinear index of silica is again $n_2 = 2.6 \times 10^{-20}$ m^2/W. The two-stage EDFA has $6+7 = 13$ dB gain in total to repeat the signal power, and the noise figure of each stage is 5 dB. The DCF has $\alpha = 0.6$ dB/km, $D = -100$ ps/nm/km, $S = -0.8$ ps/nm^2/km, $A_{\rm eff} = 25$ μm^2, but nonlinearity neglected. The transmitting and receiving ends are still the same as in the above SMF/DCF transmission. However, the four

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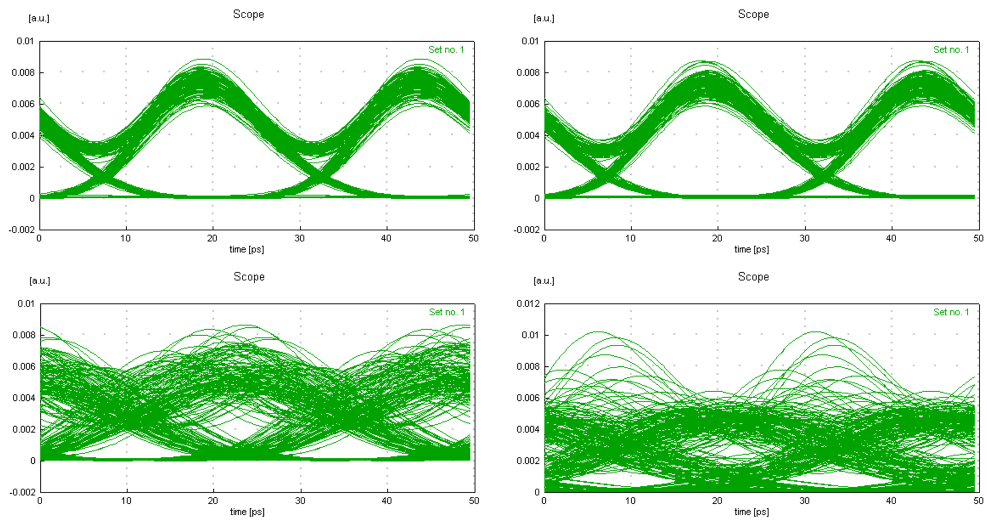

Fig. 8. Received eye diagrams of the 2nd DEMUX channel. Top row: transmission results of the setup in Fig. 7. Top-left: fiber nonlinearity is OFF, the signal is only impaired by amplifier noise. Top-right: fiber nonlinearity is ON, no extra penalty is visible. Bottom row: degraded transmission results when all -NZDSFs are replaced by +NZDSFs. Bottom-left: with OPC. Bottom-right: without OPC, of the 3rd MUX/DEMUX channel.

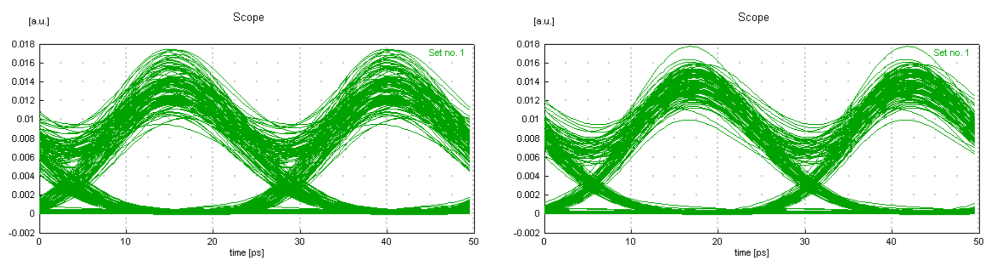

Fig. 9. Scalability and cascadability of the nonlinearity-suppressed NZDSF transmission line in Fig. 7. Left: the number of circulations on each side of OPC is doubled to ten times and the signal power is increased by 3 dB. Right: two identical transmission lines as in Fig. 7 are in cascade all-optically and the signal power is increased by 3 dB. The eye diagrams are still of the 2nd DEMUX channel.

channels of 40 Gb/s RZ pulses are transmitted at $(-350,-150,+50,+250)$ GHz off the center frequency, and they are received at $(-250,-50,+150,+350)$ GHz off the center frequency. Note that the channels are assigned asymmetrically about the center frequency to avoid phase-matched four-wave mixing (FWM) [2]. The channels may also be unequally spaced to further reduce the FWM penalty [38, 39]. But assigning channels with unequal spacing increases the network complexity and may not provide sufficient suppression by itself to the FWM and other nonlinear effects. In particular, it is ineffective to suppress the effect of cross-phase modulation (XPM). Nevertheless, when applicable, such legacy methods for nonlinearity suppression may be combined with our method of OPC-based nonlinearity compensation. The legacy methods may work to enhance the effectiveness of our method, in the sense that they may render weaker

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nonlinearity in each fiber span, so that the negligence of higher-order nonlinear perturbations becomes a better approximation. Back to the DSF-based transmission system of Fig. 10, when the power of the RZ pulses is peaked at 2 mW, Fig. 11 shows the received eye diagrams of the 2nd DEMUX channel. The top-left diagram is obtained when the fiber nonlinearity is turned OFF, so the signal is only impaired by amplifier noise. The top-right is the received eye diagram when the fiber nonlinearity is turned ON. The increased penalty due to fiber nonlinearity is visible but not too large. The eye diagrams at the bottom of Fig. 11 are obtained when the dispersion of the DCFs changes to $D = 0$ ps/nm/km while the slope remains, with or without OPC in the middle of the link. The good transmission performance shown in the bottom-left diagram verifies the insensitivity of our OPC-based method of nonlinearity compensation to the amount of residual dispersion in each fiber span, while the bad result on the bottom-right demonstrates the indispensability of OPC.

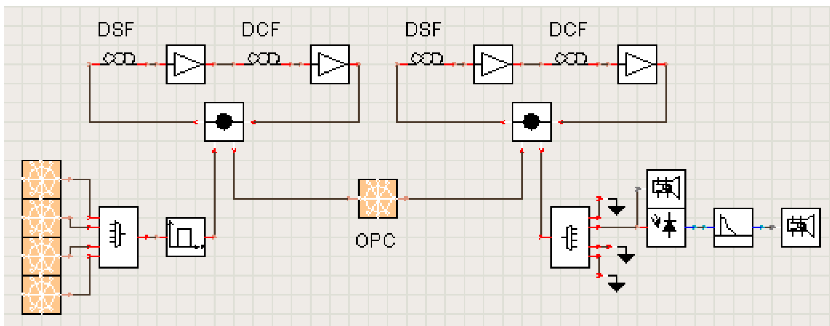

Fig. 10. A transmission line consisting of ten fiber spans on each side of OPC, each span has 50 km DSF and a slope-compensating DCF.

5. Conclusion

In conclusion, we have demonstrated through analytical derivation and numerical simulations that middle-span OPC enables simultaneous nonlinearity suppression and dispersion compensation over a wide optical band, in realistic setups of transmission lines using commercially available fibers. When the dispersion slope of transmission fibers is equalized by slope-compensating fibers, the residual dispersions of even orders are compensated by middle-span optical phase conjugation. More importantly, fiber nonlinearity is found largely suppressed, when the fiber spans are arranged into conjugate pairs about the phase conjugator, where the two fiber spans of each pair are translationally symmetric in a scaled sense. A translational symmetry about the phase conjugator is able to cancel optical nonlinearities of two fiber spans up to the first-order perturbation, while a mirror-symmetric ordering of the pairs of conjugate spans about the phase conjugator helps to prevent the accumulation of nonlinearities over a long transmission distance. One noted merit of this dual-compensation method is that its effectiveness is fairly insensitive to the amount of residual fiber dispersion after the slope compensation. The transmission fibers may be standard SMFs, NZDSFs, or even DSFs with dispersion crossing the zero point, and the slope-compensating fibers may be any DCFs with dispersion slopes opposite to that of the transmission fibers. In view of the recent developments of efficient phase conjugation based on $LiNbO_3$ waveguides [13] or highly nonlinear fibers [14], optical phase conjugators hold the promise of multiple functionalities in fiber-optic transmission systems.

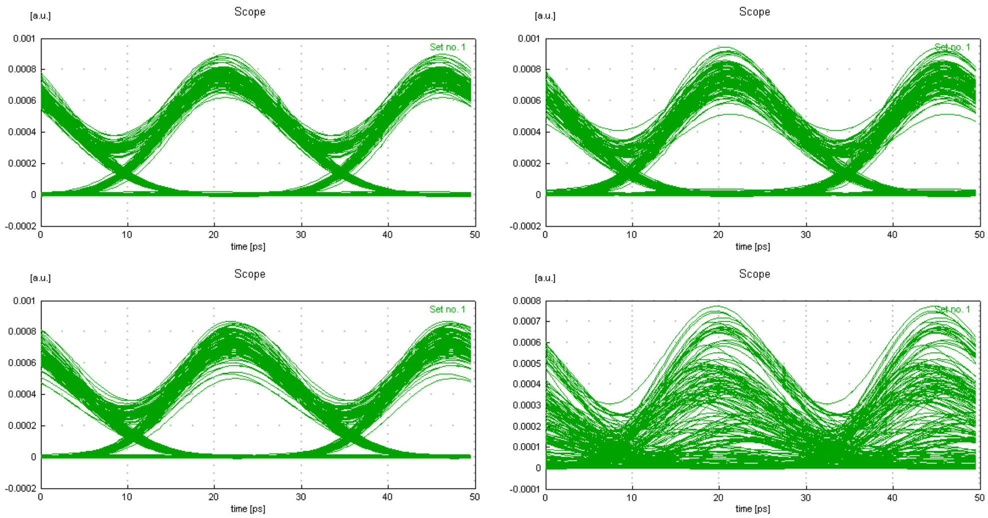

Fig. 11. Received eye diagrams of the 2nd DEMUX channel. Top row: transmission results of the setup in Fig. 10. Top-left: fiber nonlinearity is OFF, the signal is only impaired by amplifier noise. Top-right: fiber nonlinearity is ON. Bottom row: transmission results when the setup in Fig. 10 is modified by setting $D = 0$ ps/nm/km for the DCFs while keeping the dispersion slope. Bottom-left: with OPC in the middle of the link. Bottom-right: when OPC is removed.

Acknowledgments

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